Mechanical Systems and Signal Processing 107 (2018) 529–548

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Identification of multiple leaks in pipeline: Linearized model, maximum likelihood, and super-resolution localization

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ARTICLE INFO

Article history:
Received 19 June 2017
Received in revised form 21 December 2017
Accepted 29 January 2018

Keywords:
Leak detection
Defect detection
Transient wave
Maximum likelihood
Super-resolution
Cramér-Rao lower bound

ABSTRACT

This paper considers the problem of identifying multiple leaks in a water-filled pipeline based on inverse transient wave theory. The analytical solution to this problem involves nonlinear interaction terms between the various leaks. This paper shows analytically and numerically that these nonlinear terms are of the order of the leak sizes to the power two and; thus, negligible. As a result of this simplification, a maximum likelihood (ML) scheme that identifies leak locations and leak sizes separately is formulated and tested. It is found that the ML estimation scheme is highly efficient and robust with respect to noise. In addition, the ML method is a super-resolution leak localization scheme because its resolvable leak distance (approximately $0.15\lambda_{min}$, where λ_{min} is the minimum wavelength) is below the Nyquist-Shannon sampling theorem limit ($0.5\lambda_{min}$). Moreover, the Cramér-Rao lower bound (CRLB) is derived and used to show the efficiency of the ML scheme estimates. The variance of the ML estimator approximates the CRLB proving that the ML scheme belongs to class of best unbiased estimator of leak localization methods.

1. Introduction

Leaks in water supply pipelines is a problem of increasing interest due to their associated financial cost from the wastage of resources as well as their ability to act as entry points for contaminants into the treated water system. Various leak detection methods have been developed in the past decades with acoustic analysis as one of the more popular techniques. The fact that 40% of water is lost from pipes around the world is a clear testimony that current methods are far from satisfactory and there is an urgent need to fill this gap. As a result, recent researches have focused on transient-based leak detection methods (TBDMs). TBDMs utilize the hydraulics of transient flows to detect leaks in the pipeline, e.g., Refs. [1–18]. The tenet of TBDM is that leaks can be identified by injecting perturbations into a pipeline and measuring and analysing the system response (e.g., pressure head) at specified location(s). The reason that such methods are expected to work is that a leak in a pipeline system is known to result in an increased damping of the transient pressure and acts as "reflector" to the transient wave [19]. Many TBDMs have been developed by researchers and applied to water piping systems. The class of TBDM that uses the reflective property of a leak is called transient reflection based method (TRM) [2,20–23]. Another class that uses the damping property of leak is called transient damping based Method (TDM) [5]. A third class of TBDM uses both damping and reflective properties and can be found in Refs. [1,3,4,6–9,24–30]. Note that the TBDMs mainly concern transmission mains but they

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https://doi.org/10.1016/j.ymssp.2018.01.042
0888-3270/

Nomenclature

q	discharge oscillation
h	head oscillation
x^L	leak location
z^L	pipe elevation at leak
s^L	leak size
Q_0^L, H_0^L	steady-state discharge and head of leak
x^M	sensor coordinate
$\Delta\mathbf{h}$	head difference
$\mathbf{n}$	measurement noise
a	wave speed
A	area of pipeline
l	pipe length
g	gravitational acceleration
Z	characteristic impedance
R	frictional resistance
F	Darcy-Weisbach friction factor
μ	propagation function
ω	angular frequency
ω_{th}	fundamental frequency
λ_{min}	minimum wavelength
M	sensor number
N	leak number
J	frequency number
T	sample size
σ^2	variance of noise
$\log L$	log-likelihood function

Superscripts

L	leak
U	upstream node
D	downstream node
NL	no leak
SL	single leak
M	measurement
H	conjugate transpose

Acronyms

CRLB	Cramér-Rao lower bound
FIM	Fisher information matrix
FRF	frequency response function
MFP	matched-field processing
MFP(1)	MFP with one-leak model
ML	maximum likelihood
MLE	maximum likelihood estimate
MSE	mean square error
PDF	probability density function
RE	relative error
RMSE	root mean square error
SNR	signal-to-noise ratio

have been used in distribution mains [16]. Furthermore, TBDMs can detect not only leaks but also partial blockages, partially closed in-line valves, branches, etc [16,31–33]. However, there is no proof that these methods maximize signal-to-noise ratio (SNR) and there is no concerted effort in the literature to theoretically or analytically study the effect of noise on these existing methods using a probabilistic framework such as the maximum likelihood theory, although there have been attempts at evaluating the reliability of leak detection techniques with respect to noise [22]. Yet, ultimately, such methods would need to be applied in an often highly noisy environment due to traffic, turbulence, and mechanical devices.