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Short communication

A generalized leaky FxLMS algorithm for tuning the waterbed effect of feedback active noise control systems

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ABSTRACT

To tune the noise amplification in the feedback system caused by the waterbed effect effectively, an adaptive algorithm is proposed in this paper by replacing the scalar leaky factor of the leaky FxLMS algorithm with a real symmetric Toeplitz matrix. The elements in the matrix are calculated explicitly according to the noise amplification constraints, which are defined based on a simple but efficient method. Simulations in an ANC headphone application demonstrate that the proposed algorithm can adjust the frequency band of noise amplification more effectively than the FxLMS algorithm and the leaky FxLMS algorithm.

1. Introduction

Over the past decades, active control of noise and vibration has attracted considerable interests and great progress has been made [1]. Active noise control (ANC) system attenuates unwanted noise by employing the secondary sources, whose outputs are used to interfere destructively with the original primary noise [2–7] and several ANC applications have been successful, such as the ANC headphones [3,4], the ANC headrest [5] and ANC in vehicles [6]. Feedforward and feedback structures are two basic structures in ANC systems. In a feedforward structure, the unwanted primary noise is picked up by a reference sensor before it propagates to the secondary sources [7]. In a feedback structure, the ANC controller attempts to attenuate the noise without the reference signal in the feedforward structure.

A feedforward structure needs a coherent reference signal. If such a reference signal is not available, a feedback structure is an alternative option [1]. The feedback structure has its own weakness. The first is that its noise attenuation bandwidth is typically narrow, the second is that much efforts are required to guarantee the stability of the feedback structure, and the third is the waterbed effects which claim that it is theoretically impossible to have noise reduction simultaneously at all frequencies. If the noise at some frequencies is suppressed in a feedback ANC system, the noise will be increased at some other frequencies [1]. The waterbed effect is a negative consequence of using a feedback system under specific circumstances governed by the Bode integral which is kept zero over the whole frequency band [8].

Many efforts have been made to design feedback ANC systems to have both good noise reduction performance and sufficient stability. For example, Bai et al. introduced an H_∞ optimization technique to synthesize the robust controller [4]. Rafaely et al. presented an H_2/H_∞ feedback controller design method for active sound control in a headrest, where an H_2 performance criterion with H_2 and H_∞ constraints were employed and the design procedure was formulated as a convex

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programming problem using the finite impulse response (FIR) Q-parameterization and frequency discretization, and the programming problem is finally solved by using sequential quadratic programming [5].

An intuitive approach has been proposed to design the feedback controller by flattening the noise amplification due to the waterbed effect in the whole disturbance amplification frequency band, and the amplification peaks are suppressed to relatively small value for the given disturbance attenuation [9]. In the proposed method, the design of the feedback controller relies on the least squares design criteria with a MATLAB function "fdesign.arbmagnphase", which sometimes causes sub-optimal performance due to the trading off of parameters setting. Recently, a feedback ANC system using the wavelet packet filtered-x least mean square (FxLMS) algorithm is proposed by decomposing the broadband noise into several predictable band-limited sections and controlling each section independently [10]. We have proposed a simplified adaptive feedback ANC system which adopts the error signal directly as the reference signal in an adaptive feedforward control system and has advantageous of small computational load, easy implementation, and decoupling between the feedforward and feedback structures [11,12].

The methods in [4,5,9] design the feedback controller by formulating the performance specifications from practical requirements into an optimization problem with various design criteria and constraints, and then use some optimization algorithms to search the solutions. The problem with these methods is that it cannot guarantee to find a global optimal solution and abundant experience and repeated attempts are often required to obtain a satisfactory design, so they are mainly applied to offline feedback ANC system designs. The methods in [10,11] are based on the adaptive algorithms and can be applied to online feedback ANC systems, but the waterbed effect is not considered because it is not easy to merge the waterbed effect-related constraints into the adaptive algorithms.

In practical feedback ANC systems, although noise amplification must occur at some frequencies due to the waterbed effect, it is sometimes desired that the noise amplification in a certain specific frequency range is minimized while noise amplification outside this specific frequency range is acceptable. For example, in a feedback ANC headphone, while suppressing the noise at 200–1000 Hz, it is hoped that the noise amplification at the 1500–3000 Hz range is minimized while some the noise amplification at the other frequency ranges is acceptable because the residual noise in this frequency range is particularly annoying due to the specific characteristics of the primary noise and passive headphone structure.

Therefore there is a need for tuning the frequency band of noise amplification efficiently with adaptive algorithms. Based on the results in our previous papers that the leaky FxLMS algorithm can limit the gain of the adaptive filter and improve the stability of feedback ANC systems [11,12], this paper proposes an adaptive algorithm to tune the waterbed effect for feedback ANC systems, where the scalar leaky factor of the leaky FxLMS algorithm is replaced by a real symmetric Toeplitz matrix and the elements in the matrix are calculated explicitly according to the waterbed effect constraints. The proposed algorithm adjusts the noise amplification directly and effectively, thus provides more flexibility for the online realization of the feedback ANC system.

2. The proposed algorithm

2.1. Derivation of the proposed algorithm

The internal model control (IMC) based feedback ANC system is shown in Fig. 1(a), which is composed of an error sensor to measure the residual noise $e(n)$, a secondary sound source to generate the canceling signal $y(n)$ for attenuation of the primary noise $d(n)$, a reference signal $x(n)$ is filtered through $\hat{S}(z)$, a model for estimating the secondary path $S(z)$, and a control filter $W(z)$ with L coefficients such as $\mathbf{w}(n) = [w_0(n), w_1(n), \ldots w_{L-1}(n)]^T$. Here the reference signal $x(n)$ is synthesized on the basis of $e(n)$ and the secondary signal $y(n)$ filtered by $\hat{S}(z)$. Fig. 1(b) shows the simplified adaptive feedback system introduced in [11], where the reference signal comes from the error signal directly. The simplified adaptive feedback system has advantageous in computational load and ease of implementation due to the elimination of the convolution operation required in the IMC based system. The algorithm proposed in this paper can be applied to either the IMC based or the simplified feedback system, thus the two systems are not explicitly discriminated in the following derivation.

The cost function of the leaky FxLMS algorithm is [13]

$$J(\mathbf{w}) = e^2(n) + \gamma \mathbf{w}^T(n)\mathbf{w}(n) \quad (1)$$

where $\gamma \ll 1$ is a positive number, and

$$\|\mathbf{w}(n)\|^2 = \mathbf{w}^T(n)\mathbf{w}(n) \quad (2)$$

Using the gradient descent method, the update equation for $\mathbf{w}\,(n)$ becomes

$$\mathbf{w}(n+1) = (1 - \mu\gamma)\mathbf{w}(n) - \mu e(n)\mathbf{x}'(n) \quad (3)$$

where μ is the step-size and $\mathbf{x}'(n)$ is the filtered reference signal which is the linear convolution of the reference signal and impulse response of secondary path. The tap weight vector $\mathbf{w}(n)$ in Eq. (3) finally converges in mean to a biased solution [13], i.e.,

$$\lim_{n\to\infty} E\{\mathbf{w}(n)\} = (\mathbf{R}_{\mathbf{x}'\mathbf{x}'} + \gamma \mathbf{I})^{-1}\mathbf{r}_{\mathbf{dx}'} \quad (4)$$