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Modal resonant dynamics of cables with a flexible support: A modulated diffraction problem

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ABSTRACT

Modal resonant dynamics of cables with a flexible support is defined as a modulated (wave) diffraction problem, and investigated by asymptotic expansions of the cable-support coupled system. The support-cable mass ratio, which is usually very large, turns out to be the key parameter for characterizing cable-support dynamic interactions. By treating the mass ratio's inverse as a small perturbation parameter and scaling the cable tension properly, both cable's modal resonant dynamics and the flexible support dynamics are asymptotically reduced by using multiple scale expansions, leading finally to a reduced cable-support coupled model (i.e., on a slow time scale). After numerical validations of the reduced coupled model, cable-support coupled responses and the flexible support induced coupling effects on the cable, are both fully investigated, based upon the reduced model. More explicitly, the dynamic effects on the cable's nonlinear frequency and force responses, caused by the support-cable mass ratio, the resonant detuning parameter and the support damping, are carefully evaluated.

1. Introduction

Cables are widely used in suspension/stay cable bridges, power transmission lines, mooring cables etc. The past decade has witnessed extensive investigations into cable dynamics, and insightful reviews were given by Rega [1,2] and Ibrahim [3]. Irvine and Caughey [4,5], Triantafyllou [6] mainly focused on linear cable dynamics. Hagedorn et al. [7], Luongo et al. [8], and Benedettini et al. [9] investigated cable's nonlinear oscillations. Due to cable's quadratic and cubic nonlinearities, mode internal resonance is important for understanding cable's multi-modal dynamics. Two-to-one (2:1) mode interaction, was treated by Rao [10], Perkins [11], and Srinil [12]. One-to-one (1:1) resonance can be found in Pakdemirli [13], Zhao [14], and Lacarbonara [15]. For three-to-one (3:1) mode interaction, see Lacarbonara [15], Zhao [16], and Wang [17]. At the crossover points, interestingly, simultaneous activations of both 2:1 and 1:1 would occur, which were investigated by Rega [18] and Nayfeh [19].

One of the main assumptions for the above investigations is that cable's supports are of infinite mass/stiffness. This means that cable-support interactions are always assumed to be one-way. More explicitly, only the support can affect the cable while the cable's effects on the support are totally neglected.

A support with an infinite mass/stiffness is either fixed or moving ideally/freely. These two kinds of supports affect the cable dynamics quite differently from the viewpoint of wave motions [20]. Loosely, a fixed support would exactly reverse

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cable's incident waves in a diffraction manner (to satisfy the fixed boundary condition), while an ideally moving support would excite cable's wave motions and thus radiate mechanical energy into the cable. As mentioned before, in both cases, the cable's reactions on the support are assumed to be trivial.

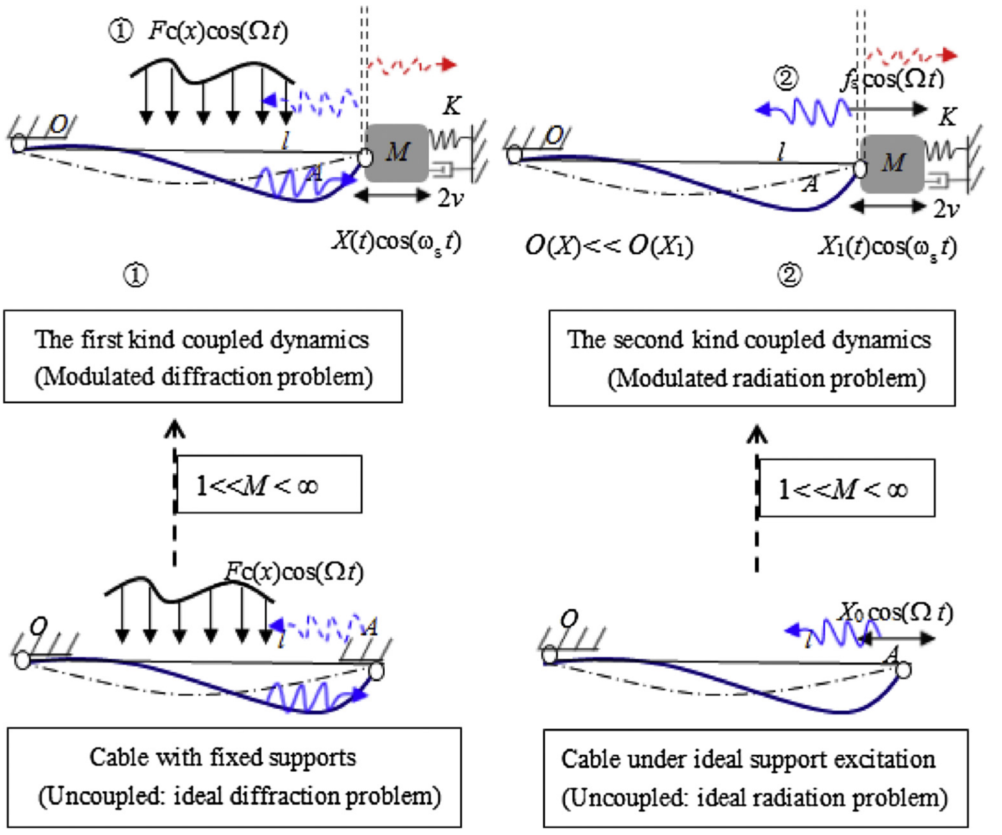
Therefore, the infinite support mass/stiffness assumption essentially leads to two distinct kinds of degenerate (or uncoupled) cable dynamics. One is cable dynamics under fixed supports and the excitations are applied on the cable itself [12–17], and the other is cable dynamics excited by ideal support motions (and/or the loadings are on the support) [21–27]. In a more broad sense, if taking the cable's vibration modes as standing waves [28], these two degenerate dynamics can be regarded as an ideal wave diffraction problem and an ideal wave radiation problem, respectively.

However, this assumption has become more and more challenged by the flexible support structures in recent years [29–36]. To understand cable dynamics with flexible support's effects, one has to abandon the infinite support mass/stiffness assumption and evaluate carefully the dynamic effects caused by the flexible support with a *finite* mass/stiffness.

In analogy with the cable's two degenerate/uncoupled dynamics mentioned above, i.e., the ideal (wave) diffraction and radiation problems, we can correspondingly categorize cable-support coupled dynamics into two different kinds, i.e., a modulated (wave) diffraction problem and a modulated (wave) radiation problem. The latter's focus is the cable's radiation dynamics induced by non-ideal support motion/excitation, which is modulated by the cable's weak reactions. This was treated in our previous work [37]. In contrast, the former's focus is the cable's diffraction dynamics with nearly fixed supports. By 'nearly fixed', we mean that a quite small support motion is caused by the cable's weak reactions, leading to slight radiation effects on the cable. One notes that the support moves even more weakly in the former than the latter cases. This is because the support motion in the modulated diffraction dynamics is caused by the cable's weak dynamics while the support motion in the modulated radiation dynamics is the primary excitation source for the cable dynamics.

All the above ideas and concepts are illustrated in Fig. 1 through comparing two different kinds of cable-support dynamics, i.e., diffraction dominated and radiation dominated. Note that the support-cable mass ratio *M* (or the support's dimensionless mass) is the key parameter for characterizing cable–support coupling intensity.

The present work concentrates on cable's modal resonant dynamics with flexible support's modulations, i.e., the modulated diffraction problem, through a cable-support coupled modeling and analysis. Our key observations are that the



(a): diffraction dominated dynamics (b): radiation dominated dynamics [37]

Fig. 1. Two different kinds of cable-support dynamics: wave motion's view.