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A probability distribution model of tooth pits for evaluating time-varying mesh stiffness of pitting gears

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ABSTRACT

Tooth damage often causes a reduction in gear mesh stiffness. Thus time-varying mesh stiffness (TVMS) can be treated as an indication of gear health conditions. This study is devoted to investigating the mesh stiffness variations of a pair of external spur gears with tooth pitting, and proposes a new model for describing tooth pitting based on probability distribution. In the model, considering the appearance and development process of tooth pitting, we model the pitting on the surface of spur gear teeth as a series of pits with a uniform distribution in the direction of tooth width and a normal distribution in the direction of tooth height, respectively. In addition, four pitting degrees, from no pitting to severe pitting, are modeled. Finally, influences of tooth pitting on TVMS are analyzed in details and the proposed model is validated by comparing with a finite element model. The comparison results show that the proposed model is effective for the TVMS evaluations of pitting gears.

1. Introduction

Gearboxes are widely used in many industrial applications such as wind energy, helicopters, and heavy trucks. However, due to heavy loads, bad lubrication and other harsh operating conditions, gears may suffer from the problem of insufficient contact fatigue strength. Damages such as pitting and spalling (a larger pit is often referred to as a spall [1]) usually occur on the surface of gear teeth [2–4]. Generally, tooth damage often causes the effective capability for bearing load decreasing and shows a reduction in gear mesh stiffness [5–7]. Therefore, the severity of tooth damage can be assessed by investigating the reduction of time-varying mesh stiffness (TVMS) in gear meshing [8–10].

Although investigations of mesh stiffness are essential for revealing the appearance and development mechanism of pitting fault [11,12], research on TVMS of pitting gears is rather limited [13,14]. Chaari et al. [3] studied the magnitude and phase changes in gear mesh stiffness caused by a single tooth pit or crack on the sun gear. Then Chaari et al. [4] modeled spalling as a rectangular indentation and studied effects of the spalling damage on the gear mesh stiffness. The similar work was carried out by Abouel-seoud et al. [15], Cheng et al. [16], and Ma et al. [17]. Bedsides, the potential energy method has been widely adopted to express the mesh stiffness of healthy and cracked gear pairs [18–21], while the utilization for pitting gears is limited. Ma et al. [22] modeled the spalling fault with a rectangular shape and calculated the TVMS. Liang et al. [14] modeled the pitting fault with multiple pits on a gear tooth and investigated the influence of different pitting severity levels

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on the gear mesh stiffness. Except for potential energy method, Rincon et al. [23] introduced a finite element model to assess the consequences of pitting on the meshing stiffness, where a single pit was modeled in an elliptical shape. Jia et al. [24] and Cooley et al. [25] also employed finite element models for mesh stiffness calculation of pitting gears. Tan et al. [26] experimentally created natural pitting under different levels of load and exhibited pitting progression with acoustic emission (AE) techniques.

The aforementioned studies mainly focus on a single pit [3,4,15–17,22], shown in Fig. 1(a) or multiple pits distributed evenly [14], shown in Fig. 1(b), which are far different from the real conditions of tooth pitting in Fig. 2. According to the American Society for Metals (ASM) handbook, pitting initiates due to a fatigue crack either at the surface of the gear tooth or at a small depth below the surface [1]. Apparently, the occurrence of surface cracks largely depends on the material defect and surface treatment of the gear tooth, which means that pits should emerge randomly, instead of a single pit or multiple pits distributed evenly. In other words, as one kind of surface faults, once the pitting occurs, no matter slight or severe, it would emerge with a series of pits on the gear tooth. Fig. 2 shows the pitting growth under different damage degrees, which was obtained from an experiment [26]. It can be seen that the aforementioned pitting models with a single small pit [3,4,22] or multiple pits distributed evenly [14] on the tooth surface are not appropriate. Besides, after the pits are generated, the faulty surface continues to mesh with other teeth. Therefore, the pits produced previously continue to propagate with the surface cracks propagating during the following operation, and the sizes of these pits continue to be increased. Therefore, the sizes of all pits are modeled as fixed or constant ones in these models [4,14,23], which is not true, either. In addition, in the former healthy zone, new pits emerge as time goes on. Therefore, the aforementioned studies modeling the pits as fixed ones have a large difference from real conditions.

Aiming to overcome the above shortcomings, a probability distribution model of tooth pits for spur gears is proposed in this paper. In the proposed model, considering the appearance and development process of tooth pitting, we model the pitting on the surface of spur gear teeth as a series of pits with a uniform distribution in the direction of tooth width and a normal distribution in the direction of tooth height. In addition, four pitting degrees from no pitting to severe pitting are studied. Among the four degrees, the sizes of pits in the former damage degree increase in the latter damage degrees, and new pits are produced in the following damage degrees as well.

The rest of this paper is organized as follows. In Section 2, the proposed pitting model based on probability distribution is presented in details, and the TVMS of pitting tooth in four pitting degrees is evaluated. A finite element model of meshing gears is established and the obtained TVMS is compared with the stiffness results using the finite element method (FEM) in Section 3. Conclusions are drawn in Section 4.

2. Model formulation for tooth pitting and TVMS evaluation

2.1. Modeling of tooth pitting

2.1.1. Distribution of pits

As mentioned in the introduction, pitting usually emerges with a series of pits due to material defects or inadequate lubrication, which means that pits emerge randomly. Therefore, the locations of pits should be modeled as random variables. In this paper, a uniform distribution function is utilized to model the pitting locations along the tooth width of a spur gear. In addition, when the contact point coincides with the pitch line, pure rolling is arisen, which is difficult to form the oil film. If the lubrication is inadequate or the load is excessive, fatigue particles from the gear pitch line may be easily created [1]. Therefore, pitting caused by surface fatigue usually occurs as a narrow band just below or at the pitch line [1], which means that in the direction of tooth depth, the distribution of pits would have a concentration below the pitch line. In this paper, we adopt the normal distribution function to describe the distribution of pits along the tooth height.

In summary, the pitting location on the gear tooth is modeled as a two-dimensional random variable in this study. If we set x_i and y_i as the coordinate values of the ith pit in the direction of tooth depth and tooth width, respectively, as illustrated in Fig. 3, there are the following relationships.

$$x_i \sim N(\mu,\ \sigma^2) \tag{1}$$

$$y_i \sim U(0,\ L) \tag{2}$$

where L is the tooth width of the spur gear. Besides, for the normal distribution function in Eq. (1), the 3σ criterion is used to describe the distribution areas of pits in this paper, which means that in the direction of tooth height, 99.73% of the pits appear in the field of $\mu - 3\sigma \leqslant x_i \leqslant \mu + 3\sigma$. Therefore, μ and σ can be calculated as follows.

$$\mu = x_{\mathrm{p}} - \delta, \sigma = \frac{x_{\mathrm{p}} - x_{\min} - \delta}{3} \tag{3}$$

where x_{p} is the coordinate value of the pitch circle in the x direction, $x_{\min}$ is determined by the boundary of meshing area, and δ indicates that the distribution of pits usually has a concentration below the pitch line, illustrated in Fig. 3.