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System equivalent model mixing

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ABSTRACT

This paper introduces SEMM: a method based on Frequency Based Substructuring (FBS) techniques that enables the construction of hybrid dynamic models. With System Equivalent Model Mixing (SEMM) frequency based models, either of numerical or experimental nature, can be mixed to form a hybrid model. This model follows the dynamic behaviour of a predefined weighted *master* model. A large variety of applications can be thought of, such as the DoF-space expansion of relatively small experimental models using numerical models, or the blending of different models in the frequency spectrum. SEMM is outlined, both mathematically and conceptually, based on a notation commonly used in FBS. A critical physical interpretation of the theory is provided next, along with a comparison to similar techniques; namely DoF expansion techniques. SEMM's concept is further illustrated by means of a numerical example. It will become apparent that the basic method of SEMM has some shortcomings which warrant a few extensions to the method. One of the main applications is tested in a practical case, performed on a validated benchmark structure; it will emphasize the practicality of the method.

1. Introduction

Structural dynamic analysis is an essential step in the design of high-tech mechanical systems. Complex products such as cars, airplanes, and high-tech machines are designed in an increasingly modular fashion, combining off-the-shelf components with newly designed parts. This generally requires the construction of dynamic models for each component in the system, which can be assembled or 'substructured' together in order to evaluate dynamic properties of the full product, such as global vibration modes or mechanical/acoustical transfer functions. Developments in Dynamic Substructuring (DS) [1–4] have increased the flexibility to combine component models from multiple modelling domains, such that experimentally obtained models may be incorporated with similar ease to numerical models. Still, the component models must fulfil two main requirements: they must correctly depict the dynamic properties of the actual component (e.g. resonance frequencies and damping) and possess clearly defined interfaces for assembling to their adjacent components.

Numerical modelling has long been the industry practice and is particularly strong in the latter: creating models with high spatial resolution from which interface degrees of freedom (DoF) are easily and unambiguously obtained. To correctly

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Nomenclature

DoF	degree of freedom
FRF	frequency response function
$\mathbf{u}$	dynamic displacements/rotations
$\mathbf{f}$	applied forces/moments
$\mathbf{g}$	interface forces/moments
$\mathbf{Y}$	admittance FRF matrix
$\mathbf{Z}$	impedance FRF matrix
$\mathbf{T}$	transformation matrix
$\mathbf{B}$	signed Boolean coupling matrix
$\mathbf{C}$	compatibility coupling matrix
$\mathbf{E}$	equilibrium coupling matrix
$\mathbf{L}$	localisation matrix
$\star^{\mathrm{par}}$	pertaining to the parent model
$\star^{\mathrm{ov}}$	pertaining to the overlay model
$\star^{\mathrm{rem}}$	pertaining to the removed model
$\star^{\mathrm{SEMM}}$	pertaining to the SEMM hybrid model
$\star^{+}$	pseudo inverse
$\star_b$	boundary or interface DoF
$\star_i$	internal DoF
$\star_d$	discarded internal DoF
$\star_k$	kept internal DoF
$(+)$	Model coupling
$(-)$	Model decoupling

represent the dynamics of the actual component, models are often updated with experimental dynamic properties obtained from measurements. Advancement in experimental techniques now also facilitate experimental modelling as an integral means to obtain dynamic models, for instance represented by frequency response functions (FRF) for all relevant interface DoF. This has led to an increase in experimental modelling of relatively complex structures, due to the fact that experimental models offer the dynamic transfers of the mechanical system ‘as is’, whereas the numerical model offers a ‘best-approximated’ description.

1.1. Difficulties & remedies in experimental modelling

Yet, standalone experimental models lack the strong suits of the numerical model. It remains challenging to extract a consistent dynamic model from essentially independent (and often imperfect) measurements, performed on a limited number of non-collocated DoF. Many strategies have been proposed to mitigate these shortcomings:

- *Modal fitting*: these techniques fit the observed dynamics (FRF) to an analytical dynamic manifold, expressed by a finite set of (linear) vibration modes with, per definition, consistent dynamic behaviour. However, these methods do not incorporate the full extent of the experimental results; this is mainly because they project all measured physical effects on a model with limited dynamic leeway [5,6].
- *Expansion using numerical models*: several techniques employ FE-models in order to ‘fill in the blanks’ between the measured nodes of the experimental FRFs. Static expansion methods like Guyan expansion use the stiffness matrix, sometimes expanded with accelerance terms as is the case with the Improved Reduction System (IRS). Other methods like Hurty Craig-Bampton, SEREP and VIKING also incorporate dynamic behaviour [7–9].
- *Expansion using local rigidness*: a typical shortcoming of experiments is a lack of rotational DoF and inability to express translational/rotational responses at the exact location where forces/moments act (sometimes called *collocated* or *vectorially-associated DoF*). The Virtual Point Transformation solves this by combining multiple translational DoF and assuming that the structure surrounding the interface exhibits rigid behaviour. In essence, this involves an expansion using six rigid Interface Displacement Modes (IDMs) per coupling point, or more if flexible interface behaviour is to be included [10–12].
- *Simulating realistic boundary conditions*: instead of trying to capture the interface dynamics in free conditions, one might also mass-load the interfaces of interest, to be closer to the assembled condition. Substructure coupling and decoupling techniques can be used to remove or replace the surrogate parts. This concept is probably best known as the Transmission Simulator method for use in the modal domain, but can be equally effective in frequency-domain substructuring [13–16].