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Eigenvalue sensitivity of sampled time systems operating in closed loop

Dionisio Bernal

Civil and Environmental Engineering Dept., Center for Digital Signal Processing, Northeastern University, Boston, MA 02115, United States

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ABSTRACT

The use of feedback to create closed-loop eigenstructures with high sensitivity has received some attention in the Structural Health Monitoring field. Although practical implementation is necessarily digital, and thus in sampled time, work thus far has center on the continuous time framework, both in design and in checking performance. It is shown in this paper that the performance in discrete time, at typical sampling rates, can differ notably from that anticipated in the continuous time formulation and that discrepancies can be particularly large on the real part of the eigenvalue sensitivities; a consequence being important error on the (linear estimate) of the level of damage at which closed-loop stability is lost. As one anticipates, explicit consideration of the sampling rate poses no special difficulties in the closed-loop eigenstructure design and the relevant expressions are developed in the paper, including a formula for the efficient evaluation of the derivative of the matrix exponential based on the theory of complex perturbations. The paper presents an easily reproduced numerical example showing the level of error that can result when the discrete time implementation of the controller is not considered.

1. Introduction

A classic problem in control theory is the selection of static gains leading to closed-loop eigenstructures with desired characteristics. In structures subjected to narrow band disturbances, for example, one seeks to keep all the close-loop poles away from the peaks of the excitation spectrum. Hardware deployed for vibration control can also be used to perform closed-loop Structural Health Monitoring (SHM) in periods when its primary function is unnecessary. In this instance the objective is not to exert control on the response but to create an eigenstructure whose sensitivity facilitates interrogation regarding the existence of damage. As one gathers, this is an application where eigenvalue sensitivity plays a central role.

The idea of doubling up the hardware to perform SHM in closed-loop was introduced by Ray and Tian [1] who recommended, based on single-degree-of-freedom considerations, that the gain be selected to shift the poles towards lower frequencies. In a subsequent examination, Jiang, Tang and Wang [2] used the fact that the expression for the closed-loop sensitivity is a function of the right and left eigenvectors and exploited the freedom in eigenvector placement offered by multiple actuators to increase sensitivity while penalizing the magnitude of the control gain. Other work on the optimization of the gain for sensitivity and some exploratory experimental work can be found in [3–8].

Studies carried out thus far on closed-loop sensitivity enhancement have been based on the continuous time (CT) framework. Practical implementations are, however, invariably digital and the question opens up as to how the digital to analog

E-mail address: bernal@neu.edu

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(D/A) conversion and the sampling rate affect the results. This paper shows that the eigenvalue sensitivities realized in the discrete time (DT) implementation can differ significantly from the CT results, even at sampling rates that can be considered fast relative to the shortest resonant period in the system. Particularly important in this regard being the fact that the discrepancy is often large on the real part of the sensitivity, and thus on the level of parameter changes (damage in the Structural Health Monitoring application) for which the closed-loop is predicted to remain stable in a linear estimate. In what follows we will refer to the eigenvalues and sensitivities that would be obtained by an ideal identification algorithm operating on the sampled data as the "realized" closed-loop eigenvalues and sensitivities. As noted, the realized properties depend on the sampling rate and on the specifics of the D/A conversion, and on this last aspect we adopt the simplest of all the options that satisfy the causality constraint, the widely used zero order hold (ZOH).

A feature in the computation of DT sensitivities that does not exist in the CT model is the need for the derivative of the matrix exponential. Included in the paper are two exact expressions for this derivative, the first in terms of an integral [9] and the second as an infinite series in terms of lie brackets [10]. Because it is somewhat tangential to the paper's objective we do not discuss the matter in detail but note that their numerical evaluation is not efficient compared to an estimation based on the theory of complex perturbations [11–13], which is thus recommended for applications. The paper derives the expressions that give the realized closed-loop sensitivity, presents a brief discussion on the linear estimation of parameter changes leading to closed-loop instability and includes an easily reproducible numerical example exemplifying the main points.

2. Effective closed-loop sensitivity

The state space recurrence in discrete time for a Linear Time Invariant system operating in closed-loop writes

$$x_{k+1} = A_d x_k + B_{d,u} u_k + B_{d,f} f_k \tag{1}$$

where $A_d \in R^{NxN}$, $B_{d,u} \in R^{Nxr}$ and $B_{d,f} \in R^{Nxq}$ are the transition, control to state, and external forces to state matrices, respectively, and x, u and f $\in R^{Nx1}, \in R^{rx1}, \in R^{qx1}$ are the state, the control forces, and the exogenous excitation, with N = system order, r = number of actuators and q = number of external actions. For a control action based on static constant gain one has

$$u_k = -Ky_k = -KCx_k \tag{2}$$

where $C \in R^{mxN}$ is the state to output matrix, with m = number of measurements, and the minus sign is, of course, conventional. In writing Eq. (2) we've assumed that the measurements do not include collocated accelerations. Substituting Eq. (2) into Eq. (1) the transition matrix in closed-loop writes

$$\bar{A}_d = A_d - B_d KC \tag{3}$$

Let θ be a parameter of the description of the system in CT, differentiating Eq. (3) with respect to θ writes

$$\bar{A}'_d = A'_d - B'_d KC \tag{4}$$

where independence of C from the parameter implies that acceleration measurements have been excluded. Given a non-defective matrix $\bar{A}_d(\theta)$ with eigenvalues $\lambda_d(\theta)$ and left and right side eigenvectors $\varphi(\theta)$ and $\psi(\theta)$ the derivative of the jth eigenvalue with respect to θ writes

$$\lambda'_j = \varphi_j^T \bar{A}'_d \psi_j \tag{5}$$

where we've left out explicit reference to the parameter to simplify the notation. The relation between B_d in Eq. (4) and its continuous time counterpart is a function of how the control action is delivered. It is common to operate on the premise that this action is applied though a D/A zero order hold circuit, which, neglecting delays, leads to the relation [14,15]

$$B_d = A_c^{-1}(A_d - I)B_c \tag{6}$$

with B_c = the control input to state matrix in continuous time. Differentiating Eq. (6) writes

$$B'_d = A_c^{-1}(A'_d B_c + A_d B'_c - B'_c - A'_c B_d) \tag{7}$$

and substituting Eq. (7) into Eq. (4) gives

$$\bar{A}'_d = A'_d - A_c^{-1}\{A'_d B_c + (A_d - I)B'_c - A'_c B_d\}KC \tag{8}$$

The open-loop transition matrix in discrete time is given by

$$A_d = e^{A_c \Delta t} \tag{9}$$

where Δt is the sampling time step. The derivative of Eq. (9) is needed to evaluate Eq. (8) and in doing so it is necessary to keep in mind that the derivative of the exponential matrix does not follow the elementary calculus rules but writes [9]