Mechanical Systems and Signal Processing 104 (2018) 1–18

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

A space-frequency multiplicative regularization for force reconstruction problems

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ARTICLE INFO

Article history:
Received 29 June 2017
Received in revised form 21 September 2017
Accepted 20 October 2017

Keywords:
Inverse problem
Force reconstruction
Multiplicative regularization
Mixed norms

ABSTRACT

Dynamic forces reconstruction from vibration data is an ill-posed inverse problem. A standard approach to stabilize the reconstruction consists in using some prior information on the quantities to identify. This is generally done by including in the formulation of the inverse problem a regularization term as an additive or a multiplicative constraint. In the present article, a space-frequency multiplicative regularization is developed to identify mechanical forces acting on a structure. The proposed regularization strategy takes advantage of one's prior knowledge of the nature and the location of excitation sources, as well as that of their spectral contents. Furthermore, it has the merit to be free from the preliminary definition of any regularization parameter. The validity of the proposed regularization procedure is assessed numerically and experimentally. It is more particularly pointed out that properly exploiting the space-frequency characteristics of the excitation field to identify can improve the quality of the force reconstruction.

1. Introduction

The characterization of dynamic forces acting on a structure remains one of the major industrial concerns to control broadband excitation sources or establish consistent excitation models for numerical simulation and design of complex structures. Unfortunately, the direct measurement of excitation forces is generally difficult or even impossible in practical situations. The basic idea to circumvent this practical limitation is to perform an indirect measurement from related accessible quantities such as displacement, velocity or acceleration fields.

Such techniques, referred to as force reconstruction problems, belong to the class of ill-posed inverse problems, meaning that the existence of a unique stable solution is not guaranteed. A possible solution to remedy this undesirable feature consists in including in the reconstruction problem some prior information on the forces to identify to constrain the space of admissible solutions. The mathematical transcription of this simple idea leads to express the inverse problem as a minimization problem, where prior information on the excitation forces is encoded in a regularization term. This regularization term can be incorporated in the formulation as an additive constraint, given rise to Tikhonov-like regularizations [1]. It should, however, be noted that a proper choice of the regularization term is crucial since it strongly conditions the quality of the reconstruction.

In general, two categories of reconstruction problem can arise in practical situations. The first one is related to the localization of excitation sources, while the second one consists in reconstructing the frequency spectrum or the time signal of

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https://doi.org/10.1016/j.ymssp.2017.10.027
0888-3270/

prelocalized sources. Regarding the localization problem, the regularization term only reflects the spatial prior information on the sources to identify. It is often expressed as the ℓ_p-norm of the desired solution vector. Such a norm is highly flexible to express one's prior knowledge on the nature of the forces to reconstruct [2], since smooth solutions are obtained for $p = 2$ [1,3,4], while sparse excitation fields are promoted for $p \leqslant 1$ [5–9]. It should be noticed that the spatial prior information is classically defined in a global manner, meaning that all the sources exciting the structure have the same attribute, i.e. localized, distributed or piecewise continuous. Such global regularization terms lead to poor reconstructions, if the actual excitation field combines sources of different nature, since the a priori has to reflect a compromise between contradictory distributions [2]. To bypass this difficulty, a group regularization term, defined from ℓ_p-norms, has been recently proposed to exploit local spatial prior information on both the nature and the location of excitation sources [10,11]. In all the aforementioned methods developed in the frequency domain, the reconstruction problem is solved frequency by frequency, which is equivalent to suppose that the frequency spectrum of the identified sources is discontinuous. This lack of spectral continuity is inherent to these approaches and can induce potential inaccuracies in the reconstructed frequency spectrum if the sources are broadband [12]. On the other hand, for reconstructing the frequency spectrum or the time signal of prelocalized sources, the regularization term has to reflect prior information on the nature of the excitation signal. Since the sources are usually broadband, the force signal exhibits a certain continuity. That is why, the corresponding regularization term is generally constructed from the ℓ_2-norm of the solution vector to identify [13–16]. Obviously, such identification methods are not suited for source localization and should fail when the locations of the potential sources does not match the actual ones. Consequently, it appears that the vast majority of the methods proposed in the literature are generally unable to consistently tackle both localization and spectral/temporal reconstruction problems at the same time. To the best of our knowledge, only a few methods have been developed to address these issues. However, they are often limited to the reconstruction of point sources or to configurations where the spatial distribution of the sources and the nature of the excitation signals share the same characteristics such as the sparsity [17–21] or address the space-time (or space-frequency) reconstruction problem in a separated manner [22].

It is thus of primary interest to simultaneously exploit both the spatial and the spectral/temporal features of excitation sources to aid the reconstruction process in finding the best possible solution. These requirements are actually satisfied by regularization terms derived from a mixed $\ell_{p,q}$-norm. A mixed norm is a matrix norm defined for any matrix $\mathbf{F}$ by the following relation:

$$\|\mathbf{F}\|_{p,q} = \left[\sum_i \left(\sum_j |F_{ij}|^p\right)^{\frac{q}{p}}\right]^{\frac{1}{q}} \quad \forall (p,q) \in]0, +\infty[^2. \tag{1}$$

Mixed $\ell_{2,q}$-norms, for $q \leqslant 1$, have revealed their suitability in signal and image recovery applications [23–26]. In the context of force identification, Rezayat et al. first derive a Tikhonov-like regularization using a regularization term based on a mixed $\ell_{2,1}$-norm to reconstruct broadband point forces [12,21].

In the present paper, an original regularization strategy is developed to solve both localization and spectral reconstruction problems within a unique framework. More precisely, the proposed approach first relies on the definition of a regularization term that properly reflects one's prior knowledge on the type (localized or distributed) of the excitation forces, as well as on the nature of the excitation signal. From a mathematical standpoint, this regularization term is constructed from the general mixed $\ell_{p,q}$-norm. Then, to derive the generic form of the identification problem, the proposed regularization term is included in the formulation as a multiplicative constraint. In doing so, a particular form of multiplicative regularization is obtained. This regularization strategy, originally developed by Van den Berg et al. [27], has several advantages compared to the more classical additive approaches, which explains its use in the present article. It has, in particular, the merit to be free from the preliminary definition of any regularization parameter. Accordingly, it is generally faster than the related Tikhonov-like regularization, but leads to similar reconstructed solutions [28]. To clearly highlight and explain the main features of the proposed regularization strategy, this article is divided into five parts. In Section 2, the reconstruction model used for deriving the space-frequency regularization is detailed. Section 3 is devoted to the description of the generic formulation of the regularization problem. Its resolution is performed from an iterative procedure introduced in Section 4. Numerical and experimental validations of the space-frequency regularization are respectively proposed in Sections 5 and 6. Obtained results point out the practical and potential interest in exploiting both spatial and spectral prior information for improving the quality of the force reconstruction. Finally, the last part of this paper introduces the theoretical basis of the possible extension of the space-frequency regularization to time domain applications.

2. Description of the reconstruction model

The definition of the space-frequency reconstruction problem requires the construction of a model describing the dynamic behavior of the structure and relating the measured vibration field to the excitation field to identify. To this end, let us consider the general situation where the studied structure is supposed linear and time-invariant. In this case, the dynamic behavior of the structure at a particular frequency f_j is completely described by the transfer functions matrix $\mathbf{H}(f_j)$. Depending on the method used to derive this transfer function matrix, two reconstruction models can be defined. Indeed, if $\mathbf{H}(f_j)$ is measured, the reconstruction model is written [29–31]: