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Analysis of piezoelectric energy harvester under modulated and filtered white Gaussian noise

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A B S T R A C T

This paper proposes a comprehensive method for the electromechanical probabilistic analysis of piezoelectric energy harvesters subjected to modulated and filtered white Gaussian noise (WGN) at the base. Specifically, the dynamic excitation is simulated by means of an amplitude-modulated WGN, which is filtered through the Clough-Penzien filter. The considered piezoelectric harvester is a cantilever bimorph modeled as Euler-Bernoulli beam with a concentrated mass at the free-end, and its global behavior is approximated by the fundamental vibration mode (which is tuned with the dominant frequency of the dynamic input). A resistive electrical load is considered in the circuit. Once the Lyapunov equation of the coupled electromechanical problem has been formulated, an original and efficient semi-analytical procedure is proposed to estimate mean and standard deviation of the electrical energy extracted from the piezoelectric layers.

1. Introduction

Advances in sensing technologies and wireless transmission facilitate the installation of large sensor networks to monitor the health of several engineered systems as well as to enable their intelligent management throughout the lifetime. At the same time, the massive implementation of wireless sensing systems also poses new technological challenges. Besides the search for energy-efficient design solutions, the development of innovative energy harvesting solutions can play a central role on the diffusion of wireless sensing systems. In this perspective, emerging technologies that aim at powering wireless sensor nodes by harvesting the energy from ambient vibrations are especially promising for large-scale implementations.

Within this framework, the analysis of piezoelectric energy harvesters under deterministic (harmonic) excitations has received several attentions so far [1,2], but the random vibrations theory can be a more general methodology to study the global electromechanical response under uncertain ambient excitations. For instance, the analytical and numerical analysis of a cantilever piezoelectric bimorph under broadband random vibration is presented in Ref. [3]. The mean power and other relevant statistics (i.e., level crossing of the voltage, response voltage peaks above certain level, fractional time of the voltage response spend above a certain level) of linear energy harvesters under broadband random excitation have been obtained in Refs. [4,5]. The hypothesis of broadband random vibration – such as the white Gaussian noise (WGN) – has proven especially useful to obtain analytical results for energy harvesters in presence of some kinds of nonlinearities [6–8]. It

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should be pointed out, however, that such assumption is an ideal simplification rather than an adequate representation of common dynamic loading conditions encountered in reality. Actually, many dynamic loads have most of their energy trapped within a narrow bandwidth. Moreover, amplitude and/or frequency content of real dynamic excitations can be subjected to significant fluctuations in time that must be properly considered for a reliable estimation of the generated energy. In this context, the stationary response of multiple piezopatches and nonlinear harvesters subjected to band-limited random vibrations has been addressed in Ref. [9] and Ref. [10], respectively. Yoon and Youn [11] have calculated the average power generated by a linear elastic bimorph under non-stationary random vibrations.

It should be also highlighted that the mean electrical energy alone provides an incomplete probabilistic appraisal in case of uncertain dynamic loadings. Because of the inherent randomness of the dynamic excitation, assessing the variability of the harvestable energy is of paramount importance as well.

Therefore, the present paper proposes a new methodology for the probabilistic analysis of a cantilever bimorph in order to evaluate the first- and second-order moments of the electrical energy extracted from the piezoelectric layers. The dynamic excitation is simulated by means of an amplitude-modulated WGN which is filtered through the Clough-Penzien filter. The linear piezoelectric generator is a classical elastic cantilever bimorph with a point mass at the free-end. It is assumed that the harvester's response under stochastic base motion is well approximated by the dynamics of the fundamental vibration mode only, which is tuned with the dominant frequency of the dynamic input. A resistive electrical load is considered in the circuit. Once the non-stationary form of the Lyapunov equation is obtained, an innovative semi-analytical procedure is proposed to estimate mean and standard deviation of the generated electrical energy. The hypothesis of linear piezoelectricity has been adopted in order to develop an attractive analytical approach, but it can lead to errors when nonlinear phenomena cannot be neglected. The statistical linearization method can be implemented to exploit the main outcomes of the present work in presence of nonlinearities, but it usually requires somewhat stringent assumptions. A numerical example concerning with a piezoelectric bimorph subjected to seismic accelerations at the base is finally presented to illustrate the application of the proposed methodology.

2. Stochastic analysis of piezoelectric energy harvester under random vibrations

2.1. Stochastic model of the dynamic excitation

A consolidated and effective stochastic modeling for random loads characterized by non-uniform frequency contents is obtained by filtering a WGN. In the context of random vibration theory, two commonly adopted linear filtering techniques are based on the Kanai-Tajimi filter and the Clough-Penzien filter [12–16]. The Kanai-Tajimi model employs a single linear second-order filter. A refinement of this stochastic modeling is based on the Clough-Penzien filter, which takes the Kanai-Tajimi filter response as the input of another linear second-order filter designated to remove pseudo-static components (thus behaving like a high-pass filter). Hence, in the Clough-Penzien model (Fig. 1a), the stochastic base motion $\ddot{x}_b$ is given as [17]:

$$\ddot{x}_b = \ddot{x}_f = -2\xi_f\omega_f\dot{x}_f - \omega_f^2 x_f - 2\xi_g\omega_g\dot{x}_g - \omega_g^2 x_g, \tag{1}$$

where x_g and x_f are the solutions of the following coupled stochastic oscillators:

$$\begin{cases} \ddot{x}_g + 2\xi_g\omega_g\dot{x}_g + \omega_g^2 x_g = -\varphi w \\ \ddot{x}_f + 2\xi_f\omega_f\dot{x}_f + \omega_f^2 x_f = -2\xi_g\omega_g\dot{x}_g - \omega_g^2 x_g \end{cases} \tag{2}$$

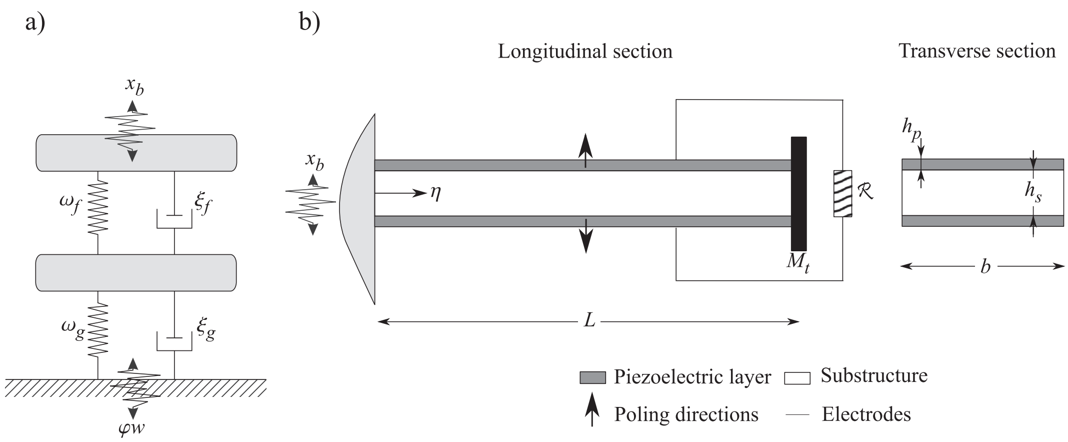

Fig. 1. Numerical models of (a) stochastic dynamic excitation and (b) bimorph cantilever with series connection of piezoelectric layers subjected to random vibrations at the fixed base.