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Brief paper

# Asynchronous decentralized event-triggered control[☆]

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ABSTRACT

In this paper we propose an approach to the implementation of controllers with decentralized strategies triggering controller updates. We consider set-ups with a central node in charge of the computation of the control commands, and a set of not co-located sensors providing measurements to the controller node. The solution we propose does not require measurements from the sensors to be synchronized in time. The sensors in our proposal provide measurements in an aperiodic way triggered by local conditions. Furthermore, in the proposed implementation (most of) the communication between nodes requires only the exchange of one bit of information (per controller update), which could aid in reducing transmission delays and as a secondary effect result in fewer transmissions being triggered.



## 1. Introduction

Aperiodic control techniques have recently gained much attention due to the opportunities they open to reduce bandwidth and computation requirements in cyber–physical system's implementations (Anta & Tabuada, 2010; Åström & Bernhardsson, 2002; Tabuada, 2007). These savings are especially relevant in the implementation of control loops over wireless channels (Araujo et al., 2011; Rabi & Johansson, 2008). In those set-ups there is not only a limited bandwidth available, but also sensor nodes may have limited energy provided by batteries. It is therefore interesting to explore approaches which may save energy expenditures at the sensors by *e.g.* reducing the number of transmissions from those sensors, or reducing the amount of time the sensor nodes need to keep their radios listening for possible communications from other nodes. While there is an extensive recent literature on event-triggered control aimed at reducing the amount of transmissions necessary to close the control loop while maintaining stability (Cervin & Henningsson, 2008; Heemels, Sandee, & van den Bosch, 2008; Lunze & Lehmann, 2010; Molin & Hirche, 2010; Stöcker, Vey, & Lunze, 2013; Wang & Lemmon, 2011), the problem of reducing listening time has received less attention (Donkers & Heemels, 2012; Mazo & Cao, 2011, 2012; Weimer, Araújo, & Johansson, 2012). Nevertheless, it is a well-known phenomena in the sensor networks community that reducing listening times has a bigger impact on the power burden than reducing transmissions (Ye, Heidemann, & Estrin, 2002). In the present paper we try to bridge this gap by proposing controller implementations focused on reducing listening times. In order to attain this goal, we propose a technique in which the sensors do not need to coordinate with each other, and therefore do not need to listen to each other. Instead, in the proposed implementation the sensors send measurements triggered by local conditions, irrespective of what the other sensors are doing, in contrast with previous work on decentralized triggering (Mazo & Tabuada, 2011). With respect to other work on decentralized or distributed event-triggered control, we do not impose any weak coupling assumptions or very restrictive dynamics, as is often the case in work on multi-agent systems (Dimarogonas, Frazzoli, & Johansson, 2012; Heemels & Donkers, 2013; Li & Lemmon, 2011; Tallapragada & Chopra, 2012; Wang & Lemmon, 2011). Note that also Wang and Lemmon (2011), Dimarogonas et al. (2012) suffer from the drawback of continuous listening. Arguably, the work closest to ours is that presented in Donkers and Heemels (2012), however restricted to the study of linear systems.

The implementation that we propose also enables the stabilization of systems employing communication packets with very small

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payload. In particular, our technique reduces the amount of payload needed to essentially one bit. To appreciate the relevance of reducing the packets payload, besides reducing power consumption (Ye et al., 2002), one must notice that a large portion of delays in communications are due to transmission delay. These transmission delays are dependent on the size of the packages transmitted, and thus reducing the payload will indirectly reduce the communication delays present in the system. Event-triggered implementations of control systems accommodate delays by making more conservative the conditions that trigger communications than those in the delay free case. Employing more conservative conditions results, in general, in more frequent transmissions of measurements. Thus, a reduction on the payload is also expected to result in a reduction on the amount of transmissions from the sensors to the controller.

The ideas in the present paper will remind the reader of dynamic quantizers for control (Liberzon, 2003) and of dead-band control (Otanez, Moyne, & Tilbury, 2002). We have, in a way, combined those ideas with recent approaches to event-triggered control stemming from Tabuada (2007) to provide a formal analysis of implementations benefiting from all those ideas. The current paper is the result of merging previous conference contributions by the authors, providing a unified analysis and removing early mistakes and imprecise statements. As such it should be seen as a more accurate and easier to follow analysis of the proposals by Mazo and Cao (2011, 2012).

## 2. Preliminaries

We denote the positive real numbers by $\mathbb{R}^+$ and by $\mathbb{R}_0^+ = \mathbb{R}^+ \cup \{0\}$. We use $\mathbb{N}_0$ to denote the natural numbers including zero and $\mathbb{N}^+ = \mathbb{N}_0 \setminus \{0\}$. The usual Euclidean ($l_2$) vector norm is represented by $|\cdot|$. When applied to a matrix $|\cdot|$ denotes the $l_2$ induced matrix norm. A symmetric matrix $P \in \mathbb{R}^{n\times n}$ is said to be positive definite, denoted by $P > 0$, whenever $x^T P x > 0$ for all $x \neq 0$, $x \in \mathbb{R}^n$. By $\lambda_m(P)$, $\lambda_M(P)$ we denote the minimum and maximum eigenvalues of $P$ respectively. A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is said to be locally Lipschitz if for every compact set $S \subset \mathbb{R}^n$ there exists a constant $L \in \mathbb{R}_0^+$ such that: $|f(x)-f(y)| \leq L|x-y|$, $\forall x, y \in S$. For a function $f : \mathbb{R}^n \to \mathbb{R}^n$ we denote by $f_i : \mathbb{R}^n \to \mathbb{R}$ the function whose image is the projection of $f$ on its $i$th coordinate, *i.e.* $f_i(x) = \Pi_i(f(x))$. Consequently, given a Lipschitz continuous function $f$, we also denote by $L_{f_i}$ the Lipschitz constant of $f_i$. A function $\gamma : [0, a[ \to \mathbb{R}_0^+$, is of class $\mathcal{K}$ if it is continuous, strictly increasing and $\gamma(0) = 0$; if furthermore $a = \infty$ and $\gamma(s) \to \infty$ as $s \to \infty$, then $\gamma$ is said to be of class $\mathcal{K}_\infty$. A continuous function $\beta : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is of class $\mathcal{KL}$ if $\beta(\cdot, \tau)$ is of class $\mathcal{K}$ for each fixed $\tau \geq 0$ and for each fixed $s \geq 0$, $\beta(s, \tau)$ is decreasing with respect to $\tau$ and $\beta(s, \tau) \to 0$ for $\tau \to \infty$. Given an essentially bounded function $\delta : \mathbb{R}_0^+ \to \mathbb{R}^m$ we denote by $\|\delta\|_\infty$ the $\mathcal{L}_\infty$ norm, *i.e.* $\|\delta\|_\infty = \operatorname{ess\,sup}_{t\in\mathbb{R}_0^+}\{|\delta(t)|\}$.

The notion of Input-to-State stability (ISS) (Agrachev, Morse, Sontag, Sussmann, & Utkin, 2008) will be central to our discussion:

**Definition 1** (*Input-to-State Stability*)**.** A control system $\dot{\xi} = f(\xi, \upsilon)$ is said to be (uniformly globally) input-to-state stable (ISS) with respect to $\upsilon$ if there exist $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}_\infty$ such that for any $t_0 \in \mathbb{R}_0^+$ the following holds:

$$\forall \xi(t_0) \in \mathbb{R}^n, \quad \|\upsilon\|_\infty < \infty,$$
$$|\xi(t)| \leq \beta(|\xi(t_0)|, t - t_0) + \gamma(\|\upsilon\|_\infty), \quad \forall t \geq t_0.$$

Rather than using its definition, in our arguments we rely on the following characterization: a system is ISS if and only if there exists an ISS Lyapunov function (Agrachev et al., 2008).

**Definition 2** (*ISS Lyapunov Function*)**.** A continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}_0^+$ is said to be an ISS Lyapunov function for the closed-loop system $\dot{\xi} = f(\xi, \upsilon)$ if there exist class $\mathcal{K}_\infty$ functions $\underline{\alpha}$, $\overline{\alpha}$, $\alpha_v$ and $\alpha_e$ such that for all $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ the following is satisfied:

$$\underline{\alpha}(|x|) \leq V(x) \leq \overline{\alpha}(|x|)$$
$$\nabla V \cdot f(x, u) \leq -\alpha_v \circ V(x) + \alpha_e(|u|). \tag{1}$$

Often we use the shorthand $\dot{V}(x, u)$ to denote the Lie derivative $\nabla V \cdot f(x, u)$, and $\circ$ to denote function composition, *i.e.* $f \circ g(t) = f(g(t))$.

Finally, we employ the following, rather trivial, result in some of our arguments:

**Lemma 3.** *Given two $\mathcal{K}_\infty$ functions $\alpha_1$ and $\alpha_2$, there exists some constant $L < \infty$ such that:*

$$\limsup_{s\to 0} \frac{\alpha_1(s)}{\alpha_2(s)} \leq L$$

*if and only if for all $S < \infty$ there exists a positive $\kappa < \infty$ such that:*

$$\forall s \in \,]0, S], \quad \alpha_1(s) \leq \kappa\alpha_2(s).$$

**Proof.** The necessity side of the equivalence is trivial, thus we concentrate on the sufficiency part. By assumption, we know that the limit superior of the ratio of the functions tends to $L$ as $s \to 0$, and therefore, $\forall \epsilon > 0, \exists \delta > 0$ such that $\alpha_1(s)/\alpha_2(s) < L + \epsilon$ for all $s \in \,]0, \delta[$. As $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ we know that in any compact set excluding the origin the function $\alpha_1(s)/\alpha_2(s)$ is continuous and therefore attains a maximum, implying that there exists a positive $M \in \mathbb{R}^+$ such that $\alpha_1(s)/\alpha_2(s) < M$, $\forall s \in [\delta, S]$, $0 < \delta < S$. Putting these two results together we have that $\forall s \in \,]0, S]$, $S < \infty$, $\alpha_1(s) \leq \kappa\alpha_2(s)$, where $\kappa = \max\{L + \epsilon, M\}$. □

## 3. Problem definition

The problem we aim at solving is that of controlling systems of the form:

$$\dot{\xi}(t) = f(\xi(t), \upsilon(t)), \quad \forall t \in \mathbb{R}_0^+, \tag{2}$$

where $\xi : \mathbb{R}_0^+ \to \mathbb{R}^n$ and $\upsilon : \mathbb{R}_0^+ \to \mathbb{R}^m$ and the full state is assumed to be measured. In particular, we are interested in finding stabilizing sample-and-hold implementations of a controller $\upsilon(t) = k(\xi(t))$ such that updates can be performed transmitting asynchronous and aperiodic measurements of each entry of the state vector. Furthermore, if possible we would like to do so while reducing the amount of transmissions. This problem can be formalized as follows:

**Problem 4.** Given system (2) and a controller $k : \mathbb{R}^n \to \mathbb{R}^m$ find sequences of update times $\{t_{r_i}^i\}$, $r_i \in \mathbb{N}_0$ for each sensor $i = 1, \ldots, n$ such that an asynchronous sample-and-hold controller implementation:

$$\upsilon_j(t) = k_j(\hat{\xi}(t)), \tag{3}$$
$$\hat{\xi}_i(t) = \xi_i(t_{r_i}^i), \; t \in [t_{r_i}^i, t_{r_i+1}^i[, \quad \forall i = 1, \ldots, n. \tag{4}$$

renders the closed-loop system:

- uniformly globally practically asymptotically stable (UGPS), i.e. satisfying that for all $\delta > 0$, there exist a controller implementation parameter $\eta(\delta)$[2] and $\beta_\delta \in \mathcal{KL}$ such that for any $t_0 \geq 0$:

$$\forall \xi(t_0) \in \mathbb{R}^n, \quad |\xi(t)| \leq \beta_\delta(|\xi(t_0)|, t - t_0) + \delta, \quad \forall t \geq t_0;$$

[2] The update times $t_{r_i}^i$ will depend on the selection of $\eta$.