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Robust signal recovery using the prolate spherical wave functions and maximum correntropy criterion ☆

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ABSTRACT

Signal recovery is one of the most important problem in signal processing. This paper proposes a novel signal recovery method based on prolate spherical wave functions (PSWFs). PSWFs are a kind of special functions, which have been proved having good performance in signal recovery. However, the existing PSWFs based recovery methods used the mean square error (MSE) criterion, which depends on the Gaussianity assumption of the noise distributions. For the non-Gaussian noises, such as impulsive noise or outliers, the MSE criterion is sensitive, which may lead to large reconstruction error. Unlike the existing PSWFs based recovery methods, our proposed PSWFs based recovery method employs the maximum correntropy criterion (MCC), which is independent of the noise distribution. The proposed method can reduce the impact of the large and non-Gaussian noises. The experimental results on synthetic signals with various types of noises show that the proposed MCC based signal recovery method has better robust property against various noises compared to other existing methods.

1. Introduction

Prolate spheroidal wave functions (PSWFs) are an important kind of functions in information and communication theory [1]. They are verified as the most energy concentrated signals in energy concentration problem, which was studied by Slepian et al. [2–4]. The energy concentration problem aims to find bandlimited functions with maximum energy in fixed time interval. PSWFs are found to satisfy the extreme conditions in the energy concentration problem [5]. As a kind of special functions, PSWFs possess many interesting properties, such as double orthogonality in both the finite time domain and the whole real axis. PSWFs are proved to be an orthogonal basis in the Paley-Wiener space [1,6], which has extensively used for a variety of physical and engineering applications.

Signal recovery is one of the most important problems in signal processing. In 1949, the classical Shannon's sampling theorem [7] first provided a theoretical guarantee for the signal recovery problem, which is the foundation of information theory. For a sequence of samples $f(kW)$ for a signal $f(t), f(t)$ can be reconstructed by the sampling theorem as follows

$$f(x) = \sum_{k\in\mathbb{Z}} f(kW)\,\mathrm{Sinc}\left(\frac{x}{W} - k\right),$$

☆ Fully documented templates are available in the elsarticle package on CTAN.

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where W is a constant. This formula is known as a cardinal series expansion (basis functions obtained by appropriate shifting and rescaling of the *sinc*-functions). Nowadays this formula still plays a central role in signal processing.

In [8,9], researchers found the relationship for *sinc*-functions and PSWFs. They showed that the *sinc*-functions can be expanded to a linear combination of PSWFs' (also namely Slepian series) [10,11], which can be used to generalize the sampling theorem associated with PSWFs. In 2009, Senay et al. [12] first applied the PSWFs based sampling theorem to signal recovery problem and achieved good performance. After that, they extended their recovery method combining with the Tikhonov regularization in [13], which used the mean square error (MSE) criterion. MSE is a widely used criterion in signal processing [14]. The algorithms associated with MSE criterion have low complexity and analytical tractability. However, MSE is a criterion based on the second-order statistics, which is suitable for the Gaussianity assumption of noises distribution. This leads to the MSE criterion based methods are sensitive to non-Gaussian noises. In fact, the realistic noises are more complicated and do not necessarily obey the Gaussianity assumption, such as impulsive noise. Once the assumption violates, the performance of the MSE criterion based recovery methods may severely decline. To overcome such limitation, we propose a novel PSWFs based signal recovery method by applying maximum correntropy criterion (MCC). MCC is important in the information-theoretic learning [15,16]. Unlike the MSE criterion, MCC is independent of the noise distribution, which has better robustness to impulsive noise and outliers than MSE. In this work, we will develop the MCC framework to the PSWFs based recovery method to recover the signals with non-Gaussian noises.

The paper is organized as follows. Section 2 introduces the σ-bandlimited signal recovery problem and some PSWFs based related works. In Section 3, we describe our proposed method and give a complete recovery algorithm. Section 4 presents the experimental results for signals with various types of noises and non-uniformly sampling signals. Some conclusions are drawn in Section 5.

2. Preliminaries and related works

In this section, we will first formulate the problem of σ-bandlimited signal recovery in this paper. Then we introduce the basic facts about PSWFs and the related works for PSWFs based signal recovery methods. Let us describe some of the notations used throughout this paper now. Vectors will be denoted as the boldface lowercase letters, i.e., $\mathbf{x}$. Matrices will be denoted by the boldface uppercase letters, i.e., $\mathbf{A}$. The i-th component of $\mathbf{x}$ is x_i and the i,j element of $\mathbf{A}$ is $(\mathbf{A})_{ij}$.

2.1. σ-bandlimited signal recovery problem

σ-bandlimited signal recovery problem aims to reconstruct a σ-bandlimited signal $x(t)$ with noise $n(t)$ by some observed samples [17]. Here, a signal $f(t)$ is said to be σ-bandlimited [17] if its Fourier transform $\mathcal{F}(f)$ is zero outside the interval $[-\sigma,\sigma]$. The Fourier transform for $f \in \mathcal{L}^1 \cap \mathcal{L}^2(\mathbb{R};\mathbb{R})$ is defined as

$$\mathcal{F}(f)(\omega) := \int_{-\infty}^{\infty} f(t)e^{\mathbf{i}t\omega}dt, \tag{2.1}$$

where $\mathbf{i}$ is the imaginary unit. Specifically, let $\{y_i\}_{i=1}^M$ is the set of the observed samples from observation signal $y(t)$ with

$$y(t) = x(t) + n(t), \quad t \in \mathbb{R}. \tag{2.2}$$

Denote the times taken the samples $\{y_i\}_{i=1}^M$ are $\{t_i\}_{i=1}^M$, where $y_i := y(t_i), i = 1,2,\ldots,M$. Our goal is to reconstruct $x(t)$ by using $\{y_i\}_{i=1}^M$.

In the following part of this section, we will introduce the related works to recover $x(t)$. We will first give an overview of the classical recovery method and then discuss the prolate spherical wave functions based recovery methods.

2.2. Sinc-functions related recovery method

For $n(t) = 0$, Strohmer et al. [18] showed that a π-bandlimited signal $x(t)$ can be uniquely represented by a set of time-shifted *sinc*-functions, i.e.,

$$x(t) = \sum_{j\in\mathbb{Z}} b_j \frac{\sin \pi(t-t_j)}{\pi(t-t_j)}. \tag{2.3}$$

Here, t_j is a time-shifted scale. $\frac{\sin \pi(t-t_j)}{\pi(t-t_j)}$ is a time-shifted *sinc*-function. The infinite coefficients $b_j, j \in \mathbb{Z}$ need to be determined. According to their theory, σ-bandlimited signal $x(t)$ in Eq. (2.2) can also be represented as a linear combination of *sinc*-functions with shifting and rescaling as follows

$$x(t) = \sum_{j\in\mathbb{Z}} b_j \frac{\sin \sigma(t-t_j)}{\sigma(t-t_j)}. \tag{2.4}$$

If t_j is $j\pi$, i.e., the integer times of π, a $2\pi B$-bandlimited signal can be represented as