Discrete-time nonlinear damping backstepping control with observers for rejection of low and high frequency disturbances

Wonhee Kim [a], Xu Chen [b], Youngwoo Lee [c,d,*], Chung Choo Chung [e], Masayoshi Tomizuka [d]

[a] School of Energy Systems Engineering, Chung-Ang University, Seoul 06974, Republic of Korea
[b] Department of Mechanical Engineering, University of Connecticut, Storrs, CT 06269-3139, USA
[c] School of Electrical and Computer Engineering, Ulsan National Institute of Science and Technology (UNIST), Ulsan 44919, Republic of Korea
[d] Department of Mechanical Engineering, University of California, Berkeley, CA 94720-1740, USA
[e] Division of Electrical and Biomedical Engineering, Hanyang University, Seoul 04763, Republic of Korea

A R T I C L E   I N F O

A B S T R A C T

A discrete-time backstepping control algorithm is proposed for reference tracking of systems affected by both broadband disturbances at low frequencies and narrow band disturbances at high frequencies. A discrete time DOB, which is constructed based on infinite impulse response filters is applied to compensate for narrow band disturbances at high frequencies. A discrete-time nonlinear damping backstepping controller with an augmented observer is proposed to track the desired output and to compensate for low frequency broadband disturbances along with a disturbance observer, for rejecting narrow band high frequency disturbances. This combination has the merit of simultaneously compensating both broadband disturbances at low frequencies and narrow band disturbances at high frequencies. The performance of the proposed method is validated via experiments.

1. Introduction

In control engineering, a robust control design for systems affected by disturbances is a fundamental issue. A number of control methods have been developed to resolve this issue. Disturbance observers (DOBs) have been widely used for servo-enhancement and robust control [1–6]. DOB based methods can effectively suppress broadband disturbances at low frequencies. In order to reject high frequency disturbances by DOB, a Q-filter with a sufficiently high bandwidth, to cover the spectrum of the main disturbance, should be used. However, the wider bandwidth of the Q-filter may cause the amplification of measurement noise at high frequencies [7]. Recently, a DOB was designed, to estimate low frequency disturbances, without a Q-filter, in the time domain [8,9]. These DOBs also require a high observer gain to estimate the high frequency disturbances.

One approach for disturbance rejection that has been extensively studied is the use of augmented observers (AOB) [10–12]. By regarding the disturbance as an augmented state, the state variables and disturbances can be estimated using measured output. It has been shown that the transfer function between the disturbance and the disturbance estimation error is in the form of a high pass filter [11], which provides the ability to estimate disturbances with frequencies below the

bandwidth of the observer. Disturbances may consist not only low frequency components but also high frequency components. In this case a high observer gain is required to estimate high frequency disturbances in AOBs. However, such gains tend to amplify measurement noise at high frequencies. Thus, it is difficult for AOB based methods to compensate for high frequency disturbances. It is typically challenging to reject high frequency disturbances in regular servo control. These disturbances are commonly in the form of induced vibrations [13,14]. To compensate for these narrow band high frequency disturbances, internal model principle (IMP) [16] based perspectives in feedback control algorithms have been investigated [17–20]. These methods are effective in cancelling narrow band disturbances, but less effective for the compensation of broadband disturbances at low frequency. The backstepping technique provides a systematic framework for the design of tracking and regulation strategies with the disturbance rejection [21–23]. However, the design of an controller for the rejection of both low and high frequency disturbances remains unsolved.

In this paper, a discrete-time backstepping control algorithm with both low frequency broadband disturbance attenuation and high frequency narrow band disturbance attenuation is proposed for tracking the desired output of a system. The proposed control method consists of two main parts: (i) a DOB for narrow band disturbance rejection at high frequencies, and (ii) a backstepping controller with an AOB for output tracking and broadband disturbance compensation at low frequencies. A discrete time DOB, which is constructed based on infinite impulse response (IIR) filters [15,24], was applied to compensate for narrow band disturbances at high frequencies. The DOB is a convenient addition to AOB-based backstepping control compared to other IMP-based control methods, as the latter can selectively reject disturbances without altering the nominal plant dynamics. With the functionality of the DOB, a plant with mixed frequency disturbances can be regarded as a nominal plant with only low frequency broadband disturbances. The discrete-time backstepping controller with the AOB is then designed for output tracking control and broadband disturbance compensation. The AOB is developed to estimate the full state, and low frequency broadband disturbances. Nonlinear damping is implemented to improve the output tracking performance in the backstepping controller. The main advantage of the proposed method is to simultaneously compensate for both low frequency broadband disturbances and high frequency narrow band disturbances. The performance of the proposed method is demonstrated by a motion control experiment using a permanent magnet synchronous motor (PMSM).

2. DOB using selective model inversion for high frequency disturbance rejection

The plant dynamics are

$$\frac{Y(z^{-1})}{U(z^{-1})} = P(z^{-1}) \tag{1}$$

where $U(z^{-1})$ and $Y(z^{-1})$ are the Z transforms of the system input, $u(k)$ and the system output, $y(k)$, respectively. Fig. 1 shows the structure of the DOB. Here, $z^{-m}P_n(z^{-1})$ is the nominal plant model that is used in model-based feedback and feedforward designs; and m is the relative degree of the nominal model.

Remark 1. In Fig. 1, $u(k)$ and $u^*(k)$ are different. $u^*(k)$ is the control input from the nonlinear damping backstepping controller, the design of which is discussed in Section 3.

The Z transforms of the disturbance, $d(k)$ and the control input, $u^*(k)$, are defined as $D(z^{-1})$ and $U^*(z^{-1})$, respectively. A block-diagram analysis gives

$$Y(z^{-1}) = G_{yd}(z^{-1})D(z^{-1}) + G_{yu^*}(z^{-1})U^*(z^{-1})$$

where

$$G_{yu^*}(z^{-1}) = \frac{P(z^{-1})}{1 - z^{-m}Q(z^{-1}) + P(z^{-1})P_n^{-1}(z^{-1})Q(z^{-1})} \tag{2}$$

$$G_{yd}(z^{-1}) = \frac{P(z^{-1})(1 - z^{-m}Q(z^{-1}))}{1 - z^{-m}Q(z^{-1}) + P(z^{-1})P_n^{-1}(z^{-1})Q(z^{-1})} \tag{3}$$

If $z^{-m}Q(z^{-1}) \approx 1$, then,

$$G_{yu^*}(z^{-1}) \approx z^{-m}P_n(z^{-1}) \tag{4}$$

$$G_{yd}(z^{-1}) \approx 0. \tag{5}$$

The above statements ((4) and (5)) indicate that disturbance, $d(k)$, are compensated in the local feedback loop, such that the overall dynamics between $u^*(k)$ and $y(k)$ is approximately equal to the nominal model $z^{-m}P_n(z^{-1})$.

If $Q(z^{-1}) \approx 0$, then,

$$G_{yu^*}(z^{-1}) = G_{yd}(z^{-1}) = P(z^{-1}) \tag{6}$$

and the DOB is disengaged from the loop. When the DOB is combined with nonlinear damping backstepping controller, it is desired that