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Short communication

Further investigation on "A multiplicative regularization for force reconstruction"

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ABSTRACT

We have recently proposed a multiplicative regularization to reconstruct mechanical forces acting on a structure from vibration measurements. This method does not require any selection procedure for choosing the regularization parameter, since the amount of regularization is automatically adjusted throughout an iterative resolution process. The proposed iterative algorithm has been developed with performance and efficiency in mind, but it is actually a simplified version of a full iterative procedure not described in the original paper. The present paper aims at introducing the full resolution algorithm and comparing it with its simplified version in terms of computational efficiency and solution accuracy. In particular, it is shown that both algorithms lead to very similar identified solutions.

1. Introduction

In a paper, recently published in MSSP, we have introduced a multiplicative regularization to tackle source reconstruction problems [1]. The proposed formulation aims at properly exploiting one's prior knowledge on the sources to identify. To this end, it is assumed that the structure is excited in N different regions by local excitation fields $\mathbf{F_i}$ of different natures (localized or distributed), while the measured vibration field $\mathbf{X}$ is supposed to be corrupted by an additive Gaussian white noise. Under these assumptions, the reconstructed excitation field $\mathbf{F_m}$ is sought as a stationary point of the functional:

$$J_m(\mathbf{F}) = \|\mathbf{X} - \mathbf{HF}\|_2^2 \cdot \sum_{i=1}^{N} \|\mathbf{L_i F_i}\|_{q_i}^{q_i}, \tag{1}$$

where

- $\mathbf{H}$ is the transfer functions matrix of the structure, which describes its dynamic behavior;
- $\mathbf{L_i}$ is a smoothing operator controlling the regularity of the solution in region i;
- q_i is a tuning parameter defined in the interval $]0, +\infty[$ and $\| \bullet \|_{q_i}$ is the ℓ_{q_i}-norm. Practically, $q_i \leqslant 1$ if the solution vector $\mathbf{L_i F_i}$ tends to be a priori sparse. On the contrary, $q_i = 2$ if the solution vector is a priori rather distributed.

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By construction, the resolution of the reconstruction problem from the multiplicative regularization defined in Eq. (1) requires the implementation of an iterative procedure. In the original paper, we have implemented an adapted Iteratively Reweighted Least Squares (IRLS) algorithm [2,3]. Basically, it consists in computing iteratively the solution of the problem by recasting the ℓ_{q_i}–norm into a weighted ℓ_2–norm. As a result, the estimated force vector $\mathbf{F}_\mathbf{m}^{(k+1)}$ at iteration k + 1 of the IRLS algorithm is sought as the stationary point of the functional (see Ref. [1]):

$$J_m^{(k+1)}(\mathbf{F}) = \|\mathbf{X} - \mathbf{HF}\|_2^2 \cdot \left\| {\mathbf{W}^{(k)}}^{1/2} \mathbf{LF} \right\|_2^2, \tag{2}$$

where $\mathbf{W}^{(k)}$ is a global diagonal weighting matrix defined from the solution computed at iteration k, namely $\mathbf{F}_\mathbf{m}^{(k)}$, and $\mathbf{L}$ is the global smoothing operator.

In the original paper, it is indicated that after some calculations $\mathbf{F}_\mathbf{m}^{(k+1)}$ is finally expressed as:

$$\mathbf{F}_\mathbf{m}^{(k+1)} = \left(\mathbf{H}^H\mathbf{H} + \alpha^{(k+1)}\,\mathbf{L}^H\mathbf{W}^{(k)}\mathbf{L}\right)^{-1}\mathbf{H}^H\mathbf{X}, \tag{3}$$

where $\alpha^{(k+1)}$ is the adaptive regularization parameter, defined such that:

$$\alpha^{(k+1)} := \frac{\left\|\mathbf{X} - \mathbf{HF}_\mathbf{m}^{(k)}\right\|_2^2}{\left\|{\mathbf{W}^{(k)}}^{1/2}\mathbf{LF}_\mathbf{m}^{(k)}\right\|_2^2}. \tag{4}$$

However, attentive readers will notice that the force vector $\mathbf{F}_\mathbf{m}^{(k+1)}$ given by Eq. (3) with $\alpha^{(k+1)}$ defined by Eq. (4) is not exactly a stationary point of $J_m^{(k+1)}(\mathbf{F})$, since the latter is obtained from Eq. (3) with $\alpha^{(k+1)}$ defined such that:

$$\alpha^{(k+1)} = \frac{\left\|\mathbf{X} - \mathbf{HF}_\mathbf{m}^{(k+1)}\right\|_2^2}{\left\|{\mathbf{W}^{(k)}}^{1/2}\mathbf{LF}_\mathbf{m}^{(k+1)}\right\|_2^2}. \tag{5}$$

However, because $\alpha^{(k+1)}$ depends explicitly on $\mathbf{F}_\mathbf{m}^{(k+1)}$, finding a stationary point of $J_m^{(k+1)}(\mathbf{F})$ requires the implementation of an iterative procedure [4–7]. Consequently, replacing Eq. (3) by an ad hoc iterative resolution allows defining the full resolution algorithm.

In the following sections, we will show that the resolution algorithm presented in the original paper is actually a simplified version of the full resolution algorithm described below. More specifically, it will be shown through numerical and experimental validations that both algorithms lead to very similar reconstructed excitation fields, while exhibiting different performances.

2. Full resolution algorithm

As explained in the introduction, the full resolution algorithm consists in replacing the calculation of $\mathbf{F}_\mathbf{m}^{(k+1)}$ from Eq. (3) by an adapted iterative process. In other words, the full resolution algorithm is composed of a main (outer) iteration corresponding to the initialization step, the calculation of the global weighting matrix $\mathbf{W}^{(k)}$ and the evaluation of the stopping criterion as defined in Ref. [1] and a nested (inner) iterative procedure to compute $\mathbf{F}_\mathbf{m}^{(k+1)}$ and $\alpha^{(k+1)}$. Consequently, this section focuses on the implementation of the nested iterative algorithm only, since the rest of the overall resolution procedure remains unchanged compared to the original paper.

2.1. Fixed point iteration

As explained previously, the aim of the nested iterative procedure is to compute $\mathbf{F}_\mathbf{m}^{(k+1)}$ so that it be a stationary point of $J_m^{(k+1)}(\mathbf{F})$. The idea here is to implement a fixed point algorithm, for which the fixed point $\mathbf{F}_\mathbf{m}^{(k+1,j+1)}$ at (inner) iteration j + 1 of the nested process and main (outer) iteration k + 1 is expressed as:

$$\mathbf{F}_\mathbf{m}^{(k+1,j+1)} = \left(\mathbf{H}^H\mathbf{H} + \alpha^{(k+1,j+1)}\,\mathbf{L}^H\mathbf{W}^{(k)}\mathbf{L}\right)^{-1}\mathbf{H}^H\mathbf{X}, \tag{6}$$

where the adaptive regularization parameter $\alpha^{(k+1,j+1)}$ writes:

$$\alpha^{(k+1,j+1)} = \frac{\left\|\mathbf{X} - \mathbf{HF}_\mathbf{m}^{(k+1,j)}\right\|_2^2}{\left\|{\mathbf{W}^{(k)}}^{1/2}\mathbf{LF}_\mathbf{m}^{(k+1,j)}\right\|_2^2}. \tag{7}$$