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Brief paper

On the optimal worst-case experiment design for constrained linear systems ☆

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ABSTRACT

The problem of experiment design for constrained linear systems with multiple inputs is addressed. A parametric model of the system is considered. The presented theoretical results provide a guideline on how to design experiments that minimize the worst-case identification error, as measured by the radius of information of the set of feasible model parameters, calculated in any norm. In addition, it is shown that an alternative, simpler approach can be employed when input constraints are symmetric and the worst-case identification error is minimized in either 1- or ∞-norm. For such cases, on the basis of the derived results, a computationally tractable algorithm for the experiment design is proposed. The presented results are valid for a general model representation, which admits the commonly used finite impulse response model as a special case. The features of the presented method are illustrated in a numerical example.

1. Introduction

The goal of optimal experiment design in system identification is to choose the input sequence to be applied to the plant, in order to maximize the information contained in the collected data and thus minimize the uncertainty on the estimated model parameters. The literature on experiment design can be divided into two categories, i.e. "probabilistic" (see e.g. Gevers, 2005; Hjalmarsson, 2005; Pronzato, 2008; Pukelsheim, 1993) and "worst-case", on the basis of the considered assumptions on the noise signal which affects the plant output.

In the worst-case framework, the information available on the noise is given just by its magnitude bound, thus moving away from any stochastic characterization. Under this assumption, Set Membership (SM) identification approaches (see e.g. Milanese, Tempo, & Vicino, 1989; Traub & Woźniakowski, 1980) are used to derive the set, named the Feasible Parameter Set (*FPS*), containing all the parameter values such that the corresponding model is able to explain the collected measurements within the assumed noise bound. In this framework, the size of the *FPS* can be considered as an indicator of the quality of the plant estimate, and it can be used as a criterion for optimal experiment design. In particular, it is of interest to design an experiment that minimizes the worst-case (i.e. for any possible noise realization) radius of the *FPS* (also called radius of information). In Bai, Tempo, and Cho (1995), the design of periodic input sequences with constrained magnitude that minimize the radius of information in 2-norm has been considered for single input, single output (SISO) systems parameterized by a finite impulse response (FIR) model. For the same class of models, in Mäkilä (1991) it was shown that the input sequence that minimizes the worst-case radius of information in 1-norm with input magnitude constraints corresponds to a Galois sequence. In Belforte and Tay (1993), in the same settings the shortest input sequences that minimize the worst-case radius of information in 1-, 2- and ∞-norm were derived. In Casini, Garulli, and Vicino (2006), a way to design input sequences that minimize the worst-case radius of information in ∞-norm for conditional SM identification was presented. Moreover, the time complexity of the worst-case experiment design in 1-norm has been considered in Dahleh, Theodosopoulos, and Tsitsiklis (1993) and Poolla and Tikku (1994). More recently, experiment design aimed at minimizing the worst-case radius of information for nonlinear systems has been investigated in Lu and Yao (2010) and Novara (2007) and for

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linear systems with quantized measurements in Casini, Garulli, and Vicino (2011, 2012).

In summary, the existing studies on experiment design aimed at minimizing the worst-case radius of information address mainly the case of SISO systems parameterized by FIR models, and only magnitude constraints on the input are considered. However, many real world systems have multiple inputs subject to other types of constraints, like input rate constraints, which often arise due to physical limitations of the actuators, as well as constraints that couple different plant inputs. In addition, in order to reduce the model complexity (and the number of parameters that need to be identified), it is often beneficial to use different parameterizations, based on basis functions, instead of FIR models. The most commonly used basis functions are the Laguerre (see Wahlberg, 1991) and Kautz (see Wahlberg, 1994) ones as well as the generalized orthonormal basis functions (see Ninness, Hjalmarsson, & Gustafsson, 1999; Van den Hof, Heuberger, & Bokor, 1995). For the mentioned cases that are not covered by the literature, we provide here new results on the optimal worst-case experiment design. In particular, we consider systems with multiple inputs subject to general convex constraints, as well as general basis function parameterizations of the system's model. For these settings, our first result provides the answer to the question of how to design input sequences that can minimize the worst-case radius of information, computed in any norm. Although the result is very general, the computational complexity of the related input design procedure becomes quickly prohibitive. Then, we demonstrate how stronger results can be obtained when the input constraints are symmetric and if either the 1- or ∞-norm radius of information is considered, leading to a computationally tractable algorithm to design the input sequence that actually minimizes the worst-case radius of information that can be achieved in the experiment. In the specific case of SISO, FIR models with only input magnitude constraints, our results correspond to the mentioned previous findings (see e.g. Belforte & Tay, 1993; Mäkilä, 1991), hence providing a generalization of the existing theory.

The paper is organized as follows. In Section 2 we introduce the notation and problem formulation. The main results are derived in Section 3, while Section 4 presents a numerical example. Finally, conclusions are given in Section 5.

2. Problem statement

We consider a multiple input, single output (MISO), strictly proper, discrete time, linear time invariant (LTI) system, with n_u inputs. At a generic time step t the measured output of the system is given by:

$$y(t) = \varphi(t)^T\theta^0 + e(t), \tag{1}$$

where ^{T} stands for the standard matrix transpose operator, $\theta^0 \in \mathbb{R}^m$ is a vector of m model parameters that describe the system, $e(t) \in \mathbb{R}$ is a term that accounts for any measurement noise, process noise or plant-model mismatch and $\varphi(t) \in \mathbb{R}^m$ is a regressor vector that depends on the applied control inputs $u(t) \in \mathbb{R}^{n_u}$ as:

$$\varphi(t+1) = A\varphi(t) + Bu(t), \tag{2}$$

where $A \in \mathbb{R}^{m\times m}$ and $B \in \mathbb{R}^{m\times n_u}$ depend on the selected model parameterization.

Remark 2.1. The results that we derive for MISO systems are also relevant for the experiment design of multiple input, multiple output (MIMO) systems, since any MIMO system can be decomposed into several MISO systems (one MISO system for each output).

Remark 2.2. Note that the system parameterization in (1) and (2) is slightly different from the standard regression form model assumed in the system identification literature (see e.g. Gevers, 2005; Hjalmarsson, 2005; Pronzato, 2008; Pukelsheim, 1993). We use such a model structure in order to state the results on the worst case experiment design for a broad range of basis function parameterizations. In fact the chosen model structure covers as special cases several commonly used model parameterizations. For example, for the case when $n_u = 1$ and an FIR plant model is used, A and B would have the following structure:

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{3}$$

For the case when $n_u > 1$, A and B can be obtained by block diagonalizing the matrices in (3). Moreover, suitable A and B matrices can be derived for the case when the Laguerre (see Dumont & Fu, 1993), Kautz (see Wahlberg, 1994) or generalized basis functions (see Ninness et al., 1999) are used.

Remark 2.3. According to the model in (1) and (2), the regressor vector depends on the past plant inputs only. The extension of the presented results to a model in which the regressor vector would depend on the past values of the plant output (e.g. ARX models) would not be straightforward. Such models are treated in the category of errors-in-variables problems in the SM literature (see e.g. Milanese, Norton, Piet-Lahanier, & Walter, 1996).

We consider the following assumption on the matrices A and B.

Assumption 1. The pair A, B is controllable and A has all the eigenvalues inside the unit circle.

Note that Assumption 1 holds for all standard basis function parameterizations. The only knowledge that is assumed on the signal $e(t)$ is that its magnitude is bounded point-wise in time.

Assumption 2.

$$|e(t)| \le \epsilon, \quad \forall t, \tag{4}$$

where $\epsilon \ge 0$ is a known constant.

Note that in practice it might be easier to estimate the bound on the magnitude of the signal $e(t)$ than to estimate its statistical properties. The magnitude bound of the measurement noise could for example be obtained from the sensor technical specifications.

We assume that the plant inputs are subject to constraints of the form:

$$\begin{aligned} &C^T u(t) \le g, \quad \forall t \\ &L^T \Delta u(t) \le f, \quad \forall t, \end{aligned} \tag{5}$$

where $\Delta u(t) = u(t) - u(t-1)$ is the rate of change of the input. The element-wise inequalities in (5) define convex sets through the matrices $C \in \mathbb{R}^{n_u\times n_g}$ and $L \in \mathbb{R}^{n_u\times n_f}$ and the vectors $g \in \mathbb{R}^{n_g}$ and $f \in \mathbb{R}^{n_f}$, where n_g and n_f are the number of linear constraints on the input magnitudes and on their rates, respectively. In addition, we consider the following assumption on the input constraints.

Assumption 3. The set described by the input constraints (5) is compact and contains the origin.

Note that just magnitude and rate constraints have been assumed in order to simplify the notation, however all the results hold also for additional polytopic input constraints satisfying Assumption 3.

The actual value of the parameter vector θ^0 is unknown and needs to be estimated on the basis of the assumptions on the system model and on the magnitude bound of the signal $e(t)$, together