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Impact adding bifurcation in an autonomous hybrid dynamical model of church bell

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ABSTRACT

In this paper we present the bifurcation analysis of the yoke-bell-clapper system which corresponds to the biggest bell “Serce Lodzi” mounted in the Cathedral Basilica of St Stanislaus Kostka, Lodz, Poland. The mathematical model of the system considered in this work has been derived and verified based on measurements of dynamics of the real bell. We perform numerical analysis both by direct numerical integration and path-following method using toolbox ABESPOL (Chong, 2016). By introducing the active yoke the position of the bell-clapper system with respect to the yoke axis of rotation can be easily changed and it can be used to probe the system dynamics. We found a wide variety of periodic and non-periodic solutions, and examined the ranges of coexistence of solutions and transitions between them via different types of bifurcations. Finally, a new type of bifurcation induced by a grazing event - an “impact adding bifurcation” has been proposed. When it occurs, the number of impacts between the bell and the clapper is increasing while the period of the system’s motion stays the same.

1. Introduction

Bells have been musical and ceremonial instruments with a very long history and well established role in a culture. They were invented in China and have been used all around the world [8]. Today, their sound announces and upgrades significance of major events. Depending on the region bells are mounted in a number of different ways basing on local customs and tradition. In Europe we have three different characteristic mounting layouts: Central European, English and Spanish [9]. In Central Europe, bells usually tilt on their axis with maximum amplitude of oscillations below 90 degrees. In the English, system the amplitude of oscillations is greater and bells perform nearly a complete rotations in both directions. Conversely, in the Spanish system bells rotate continuously in the same direction. All these mounting layouts were developed throughout centuries based on experience and intuition of bell-founders and craftsmen. It is common that the bells are casted using casting moulds passed down from father to son and so forth. Although the design of a bell, its yoke, clapper and a belfry has been being improved over the years, their modelling and dynamical analysis is still a challenging task.

The dynamics a yoke-bell-clapper system is complex and difficult to analyze due to its nonlinear characteristics, repetitive impacts and piecewise smooth nature of its excitation. In 19th century Wilhelm Veltmann made a first attempt to describe mathematically the behaviour of the famous Emperor’s Bell in the Cologne Cathedral [27,28] by using a double

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pendulum to model the bell and the clapper. Heyman and Threlfrall [10] used a similar model to estimate inertia forces induced by a swinging bell. The important issue during yoke design is the knowledge of loads created by ringing bells. This can significantly improve the overall dynamics and reliability of the system [23,15,1]. The analysis of bells' sound is always performed via Finite Elements Methods [24,12,7], while in modelling of their dynamics, there is a tendency to use much simpler hybrid dynamical models of the yoke-bell-clapper system, e.g.[16]. This model has been improved in our recent paper [19] based on the experimental studies where parameters' values were determined from the measurements of the biggest bell in the Cathedral Basilica of St Stanislaus Kostka, Lodz, Poland. Then, in [20] we focused on the description of different ringing schemes and their coexistence of stable solutions. In the very recent work [3], we show the method to prepare the model of the yoke-bell-clapper system with external excitation for continuation in ABESPOL toolbox [4] and we show the bifurcation diagram for varying excitation's torque.

In this paper, we present the bifurcation analysis taking the yoke geometry as a bifurcation parameter. Let us first briefly discuss possible scenarios of stabilization and destabilization of periodic solutions in the context of non-smooth dynamics, which has witnessed a rapid development recently. The bifurcation theory of smooth systems is mature and we know all bifurcations with co-dimension 1 and 2 [32,13]. In the smooth systems we can distinguish the following typical local bifurcations: period doubling, Neimark-Sacker, pitchfork and saddle-node. The first three bifurcations cause destabilisation of the current periodic solution and emergence of new a periodic solution with different features. The exception is the saddle-node bifurcation, which takes place when stable and usable orbits collide and annihilate each other. In the non-smooth systems, we observe all aforementioned bifurcations and additionally grazing, period adding, corner and sliding bifurcations [2,6,18,21,25,14]. Nevertheless, we still do not have a full catalogue of non-smooth bifurcations. In our investigations, we have focused on the grazing bifurcation [31,22,29,11] and grazing induced bifurcations [17,30]. Grazing bifurcations may induce different events such as: a sudden loss of stability or existence of the orbit, a creation of new periodic orbit or multiple orbits, a change in the period of the system's motion or creation of a chaotic attractor. Most of known non-smooth bifurcations have been detected in the simple systems, where one can perform analytical investigations. However, there is a large group of complex systems which cannot be analysed analytically and for which one can expect new non-smooth bifurcations. The main reason for that is the lack of easy, accessible tools to analyse the complex non-smooth systems via path-following. Recently, new path-following toolboxes TC-Hat [26], Coco [5] and ABESPOL [4] have been developed. Thanks to such software the analysis of non-smooth systems is now possible even in complex cases like yoke-bell-clapper system, where we have multiple nonlinearities.

In this paper, we consider the application of the active yoke which let us change the position of the bell-clapper system in respect to the axis of rotation of the yoke. We show its influence on the dynamical response of the system and present the existence of several periodic and non-periodic solutions, the ranges of coexistence of solutions and transitions between them via different bifurcation scenarios. Finally, we introduce a new kind of bifurcation induced by a grazing event - an "impact adding bifurcation".

The paper is organized as follows. In Section 2 we describe the hybrid dynamical model of the church bell and introduce the active yoke. The results of the path-following are shown in Section 3. In Section 4 the conclusions are drawn.

2. Physical and mathematical models

The hybrid dynamical model of the yoke-bell-clapper system considered in this paper has been described in detail in our previous publication [19]. To calibrate the model and determine its parameters we have performed detailed measurements of the bell named The Heart of Lodz (the biggest bell in the Cathedral Basilica of St Stanislaus Kostka in Lodz). The model was then tuned and validated by comparing the results of numerical simulations with the data collected during a series of experiments proving to be a reliable predictive tool and capable to simulate the behaviour of a parameters. The next subsections briefly describe the model and present the influencing parameters.

2.1. Geometry of the yoke-bell-clapper system

The developed mathematical model is based on the analogy between freely swinging bell and the motion of the equivalent double physical pendulum. The first pendulum has fixed axis of rotation and models the yoke together with the bell that is mounted on it. The second pendulum is attached to the first one and imitates the clapper. Fig. 2.1(a) and (b) shows schematics indicating the position of the rotation axes of the bell o_1, the clapper o_2 and presenting parameters involved in the model. For the sake of simplicity, henceforth, the term "bell" is used for the bell and its yoke, which is treated as one solid element.

The model has eight physical parameters: L_0 describes the distance between the rotation axis of the bell and its centre of gravity (point C_b), l is the distance between the rotation axis of the clapper and its centre of gravity (point C_c). The distance between the bell's and the clapper's axes of rotation is given by l_{c0}. The mass of the bell is described by M, while B_{b0} characterizes the bell's moment of inertia referred to its axis of rotation. Similarly, m describes the mass of the clapper and B_c stands for the clapper's moment of inertia referred to its axis of rotation. Parameter l_r is used to describe the modifications of the yoke, as it is presented in Fig. 2.1(b) and (c). The l_r definition is explained in detail in our previous paper [20], where $l_r = 0$ refers to the shape of original yoke used in the Cathedral's bell. If the centre bell's of gravity is lowered with respect to