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Increasing accuracy in the interval analysis by the improved format of interval extension based on the first order Taylor series

Yi Li *, Yan Long Xu

School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, Shaanxi, China

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A B S T R A C T

When the dependence of the function on uncertain variables is non-monotonic in interval, the interval of function obtained by the classic interval extension based on the first order Taylor series will exhibit significant errors. In order to reduce theses errors, the improved format of the interval extension with the first order Taylor series is developed here considering the monotonicity of function. Two typical mathematic examples are given to illustrate this methodology. The vibration of a beam with lumped masses is studied to demonstrate the usefulness of this method in the practical application, and the necessary input data of which are only the function value at the central point of interval, sensitivity and deviation of function. The results of above examples show that the interval of function from the method developed by this paper is more accurate than the ones obtained by the classic method.

1. Introduction

Uncertainty quantification technologies can be broadly categorized as probabilistic or non-probabilistic methods [1]. In probabilistic approaches, the probability density functions of the uncertain parameters have to be properly defined and can be quantified by several techniques such as the Monte Carlo Simulation [2] (MCS), perturbation method [3], Neumann expansion [4], Polynomial Chaos Expansion [5] and stochastic collocation [6]. In non-probabilistic approaches the interval with upper and lower bounds can be used to represent the uncertainty range, and the uncertain parameters can be quantified by the classic interval analysis [7]. However, the application of classic interval analysis to practical engineering problems is quite difficult because it may lead to large overestimation due to the so-called dependency phenomenon [8]. In order to limit the overestimation, the generalized interval analysis [9] and the affine arithmetic [10] have been introduced by researchers. In the antenna array analysis, Poli used the interval analysis to investigate the tolerances on the power patterns radiated by linear antenna arrays whose excitations are affected by phase erros [11], Anselmi employed the interval analysis to optimize these excitation tolerances [12], and then Tenuti proposed an innovative approach based on the integration of the interval analysis and the Minkowski sum to predict tight, reliable, and inclusive pattern bounds for planer arrays [13]. And recently, within the framework of static structural analysis, the improved interval analysis by the Extra Unitary Interval (EUI) [14], Interval Perturbation Method (IPM) [15], the Parameterized Interval Analysis (PIA) [16] and Taylor Expansion (TE) [17,18] have been developed. Specifically, Muscolino and Sofi [19] combined the Interval Rational Series Expansion (IRSE)

* Corresponding author.
E-mail address: liyi504@nwpu.edu.cn (Y. Li).

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with the Improved Interval Analysis by the Extra Unitary Interval (IIA-EUI) to quantify the uncertain parameters in static analysis of structural systems. Meanwhile, Elishakoff and Miglis [20] combined the PIA with the optimization and anti-optimization problems (OAP) to solve the static structural problem with uncertain parameters. Then, Santoro et al. [21] combined the PIA and OAP with the IRSE to overcome the limits of IIA-EUI and IRSE, and efficiently solved the static problem with uncertain parameters.

In many interval analysis methods, the Taylor Expansion [17,18,22,23] is a popular one. It can give the bounds of function based on the sensitivities with respect to the uncertain variables and the deviations. It is not necessary to modify the govern function, and easy to use. Recently Wu and Zhen used the Taylor Expansion to carry out the interval uncertain optimization [24]. However, if the dependence of function on the uncertain variable is non-monotonic in interval, the error of the predicted interval of response function will be increased.

In order to limit the errors, the improved formats of the first order Taylor expansions are proposed in this paper, and these formats are chosen according to the monotonicity, positions of the central point relative to stationary point, boundary values of function and response of function at the stationary point, and then the accurate interval of function can be obtained by updating the deviations and sensitivities in the classic interval extension. To demonstrate that, two mathematic examples are studied, one is function of one variable, the other is function of two variables. And an interval analysis procedure based on the improved format interval extension and finite element method is proposed to deal with the practical problem, and a vibration problem of beam with lumped masses is studied to illustrate this procedure.

2. Error of the interval induced by the non-monotonicity

When the dependence of function on the uncertain variables is non-monotonic in interval, the real bounds are far away from the ones predicted by the classic interval extension. The error of the interval analysis is increased. In the following part of this section the sources of errors are discussed in categories.

First, the basic theory of the classic interval extension with the first Taylor series is introduced. In the interval analysis uncertain parameter can be described by interval variable

$$x^I = [\underline{x}, \bar{x}] \tag{1}$$

where $\underline{x}$ and $\bar{x}$ define the lower and upper bounds of interval. Alternatively an interval x^I can be represented by its central value and deviation, i.e.,

$$x_c = \frac{\underline{x} + \bar{x}}{2} \tag{2}$$

$$\Delta x = \frac{\bar{x} - \underline{x}}{2} \tag{3}$$

$$x^I = x_c + \Delta x e \tag{4}$$

where $e = [-1, 1]$, and the first Taylor expansion of function can be written

$$\begin{gathered} f(X^I) = f(X_c) + \sum_{i=1}^{n} \frac{\partial f(X_c)}{\partial x_i}(x_i^I - x_c) = f(X_c) + \sum_{i=1}^{n} \frac{\partial f(X_c)}{\partial x_i} \Delta x_i e_i \\ X^I = x_1^I, x_2^I, \ldots, x_n^I, \quad X_c = x_{1c}, x_{2c}, \ldots, x_{nc}, \quad e_i = [-1, 1] \end{gathered} \tag{5}$$

where $f(X_c)$ is the response at the central values of interval variables and $\frac{\partial f(X_c)}{\partial x_i}$ are sensitivities of response (at the central values) with respect to the interval variables.

By making use of the classic interval extension, the bounds of function can be obtained as follows, respectively

$$\overline{f(X^I)} = f(X_c) + \sum_{i=1}^{n} \left| \frac{\partial f(X_c)}{\partial x_i} \right| \Delta x_i, \quad i = 1, 2, \ldots, n \tag{6}$$

and

$$\underline{f(X^I)} = f(X_c) - \sum_{i=1}^{n} \left| \frac{\partial f(X_c)}{\partial x_i} \right| \Delta x_i, \quad i = 1, 2, \ldots, n \tag{7}$$

When the relationship of the function respect to uncertain design variable is monotonic, the bounds of function are extended by subtracting or adding a linear polynomial from the function value at the central point of interval, and the predicted interval of function is close to the real one, as shown in Eqs. (6)–(8) and Fig. 1.

$$\begin{aligned} &\text{Real interval}: \; [f(x_c - \Delta x), \; f(x_c + \Delta x)] \\ &\text{Predicted interval}: \; [f(x_c) - |f'(x_c)|\Delta x, \; f(x_c) + |f'(x_c)|\Delta x] \end{aligned} \tag{8}$$