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Comparison of least squares and exponential sine sweep methods for Parallel Hammerstein Models estimation

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ABSTRACT

Linearity is a common assumption for many real-life systems, but in many cases the nonlinear behavior of systems cannot be ignored and must be modeled and estimated. Among the various existing classes of nonlinear models, Parallel Hammerstein Models (PHM) are interesting as they are at the same time easy to interpret as well as to estimate. One way to estimate PHM relies on the fact that the estimation problem is linear in the parameters and thus that classical least squares (LS) estimation algorithms can be used. In that area, this article introduces a regularized LS estimation algorithm inspired on some of the recently developed regularized impulse response estimation techniques. Another mean to estimate PHM consists in using parametric or non-parametric exponential sine sweeps (ESS) based methods. These methods (LS and ESS) are founded on radically different mathematical backgrounds but are expected to tackle the same issue. A methodology is proposed here to compare them with respect to (i) their accuracy, (ii) their computational cost, and (iii) their robustness to noise. Tests are performed on simulated systems for several values of methods respective parameters and of signal to noise ratio. Results show that, for a given set of data points, the ESS method is less demanding in computational resources than the LS method but that it is also less accurate. Furthermore, the LS method needs parameters to be set in advance whereas the ESS method is not subject to conditioning issues and can be fully non-parametric. In summary, for a given set of data points, ESS method can provide a first, automatic, and quick overview of a nonlinear system than can guide more computationally demanding and precise methods, such as the regularized LS one proposed here.

1. Introduction

Systems are generally assumed to behave linearly and in a noise-free environment. This is in practice never perfectly the case. First, nonlinear dynamic behaviors are very common in real life systems [12,15]. Second, the presence of noise is a natural phenomenon that is unavoidable for all experimental measurements. In order to perform reliable model estimation of such systems, one should thus keep in mind these two issues and take care about them. Indeed, all the noise that is not correctly removed from the measurements could be misinterpreted as nonlinearities, thus polluting measurements. And if nonlinearities are not accurately estimated, they will end up within the noise signal and information about the system under study will be lost.

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The problem addressed here is the estimation of nonlinear models of real life systems [20,10,5]. In order to build a nonlinear model, some approaches are based on a physical modeling of the structure whereas some perform without any physical assumption (black-box models). As nonlinear mechanisms in structures are complex and various and as it is not intended here to build a model for each case, it is chosen to rely on black-box models. Among these black-box approaches, some assume a given form for the selected model (block-oriented models [2,10,5,28]) whereas some do not put constraints on the model structure. Because block-oriented models can be interpreted easily, this class of models has been retained. A class of block-oriented models that is particularly interesting is the class of Parallel Hammerstein Models (PHM, see Fig. 1). It belongs to the class of "*Sandwich models*" and is shown to possess a reasonable degree of generality [20,5].

In a PHM, each branch is composed of a nonlinear static polynomial element followed by a linear one, $h_n[k]$, as shown in Fig. 1 where k denotes the discrete-time in samples. The relation between the input $e[k]$ and the output $s[k]$ of such a system is given by Eq. (1), where $*$ denotes the convolution product:

$$y[k] = h_0 + \sum_{n=1}^{N} h_n * u^n[k]. \tag{1}$$

In this model, each impulse response $h_n[k]$ is convolved with the input signal raised to its nth power and the output $s[k]$ is the sum of these convolutions. h_0 stands for the constant offset. The first impulse response $h_1[k]$ represents the linear response of the system. The other impulse responses $\{h_n[k]\}_{n\in\{2\cdots N\}}$ model the nonlinearities. The family $\{h_n[k]\}_{n\in\{1\cdots N\}}$ will be referred to as the kernels of the model. The focus is put here on the estimation of the kernels of PHM. In terms of real life applications, PHMs have already proven their usefulness in modeling power amplifiers [11,8], loudspeakers [24], damaged structures [17,23], or digital audio effects [1].

One way to estimate PHM relies on the fact that the estimation problem is linear in the parameters and thus that classical least squares (LS) estimation algorithms can be used. A popular approach is thus to pass the input through a parallel connection of some nonlinear basis functions that are each followed by a finite impulse response (FIR) filter [8]. The corresponding multiple-input single-output problem is then solved using standard FIR identification method. For example, Gallman [9] used Hermite polynomials as orthogonal basis functions with Gaussian inputs. The main drawbacks of these approaches is that the user gets no physical insight in the number of parallel branches in the system under test, and that for systems with long memories a large number of parameters is needed due to the FIR-nature of the model. To limit these drawbacks Schoukens et al. [30,29] proposed to couple a best linear approximation [22] at different excitation levels with a singular value decomposition and a rational transfer function parametrization LTI subblocks. An alternative method relies on Volterra series analytical method and wavelet balance method under multilevel excitations [3]. In that area, this article introduces a regularized LS identification algorithm inspired on the recently developed regularized impulse response estimation techniques [21,14].

Another means to estimate PHM consists in using exponential sine sweeps (ESS) based methods. This idea for the analysis of nonlinear system has been first presented by Farina [6] and is sometime referred to as the "*nonlinear convolution*" [7]. This method has been later formally demonstrated [19,18,24] and it is now well established that almost nonparametric versions of PHM can be very easily and rapidly estimated using it. However, one drawback of these ESS based methods is that the number of parallel branches is chosen arbitrarily. Nevertheless, this drawback has been overcome by Rébillat et al. [25] making this estimation method now fully nonparametric.

These two classes of methods (LS and ESS) are founded on radically different mathematical backgrounds but have been developed to solve the same issue. A methodology is thus proposed here to compare them with respect to (i) their accuracy, (ii) their computational cost, and (iii) their robustness to noise. Tests are performed on simulated systems for several values of methods respective parameters and of signal to noise ratio. The paper is organized as follows: the regularized least squares method and the exponential sine sweep method are introduced in Sections 2 and 3. The systems under study and the methodology proposed to compare both methods are presented in Section 4. Results are then provided in Section 5 before concluding in Section 6.

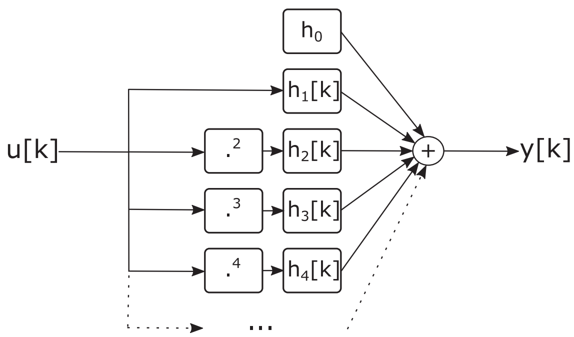

Fig. 1. Representation of Parallel Hammerstein Models (PHM).