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Efficient multidimensional regularization for Volterra series estimation

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ABSTRACT

This paper presents an efficient nonparametric time domain nonlinear system identification method. It is shown how truncated Volterra series models can be efficiently estimated without the need of long, transient-free measurements. The method is a novel extension of the regularization methods that have been developed for impulse response estimates of linear time invariant systems. To avoid the excessive memory needs in case of long measurements or large number of estimated parameters, a practical gradient-based estimation method is also provided, leading to the same numerical results as the proposed Volterra estimation method. Moreover, the transient effects in the simulated output are removed by a special regularization method based on the novel ideas of transient removal for Linear Time-Varying (LTV) systems. Combining the proposed methodologies, the nonparametric Volterra models of the cascaded water tanks benchmark are presented in this paper. The results for different scenarios varying from a simple Finite Impulse Response (FIR) model to a 3rd degree Volterra series with and without transient removal are compared and studied. It is clear that the obtained models capture the system dynamics when tested on a validation dataset, and their performance is comparable with the white-box (physical) models.

1. Introduction

In the field of system identification, modelling of nonlinear systems is one of the most challenging tasks. One possibility to model nonlinear dynamics in a nonparametric way is by means of the Volterra series [1]. The use of the series can be quite beneficial when precise knowledge about the exact nature of the nonlinear system behaviour is absent. Moreover, it has been shown in [2] that a truncated version of the infinite Volterra series can approximate the output of any nonlinear system up to any desired accuracy as long as the system is of fading memory. Practically speaking, it is sufficient that the influence of past input signals to the current system output decreases with time, a property that is met quite often in real systems, such as the cascaded water tanks system considered in this paper.

However, it is quite often possible that an accurate nonparametric fitting of nonlinear dynamics requires an excessive number of model parameters. Under this condition, either long data records should be available, resulting in computationally heavy optimization problems and long measurements, or the estimated model parameters will suffer from high variance. This is the reason why the series has in general been of limited use (for echo cancellation in acoustics [3,4] and for physiological systems [5]), and mostly in cases where short memory lengths and/or low dimensional series for the modelling

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process were sufficient (e.g. [6]). In the field of mechanical engineering numerous applications of the Volterra series can be found (e.g. for mechanical system modelling [7,8,9], for damage detection [10,11]). We refer to [12] for an extended survey on the Volterra series and its engineering applications.

In this paper, we present a method to estimate efficiently finite Volterra kernels in the time domain without the need of long measurements. There are several studies available in the literature where methods are proposed to reduce the dimensionality (memory) issues of the Volterra series estimation problem, for example, with the use of orthonormal basis functions [13,14] or tensor decompositions [15]. The main disadvantage of these techniques lies in the choice of the poles of the basis functions. The latter issue is even more involved in the case of expanding a Volterra series model with basis functions [14].

The method presented in this work is based on the regularization methods that have been developed for FIR modelling of Linear Time-Invariant (LTI) systems [16], while results exist also for the case of Frequency Response Function (FRF) estimation [17]. In the aforementioned studies, the impulse response coefficients for a LTI system are estimated in an output error setting using prior information during the identification step in a Bayesian framework. The knowledge available a priori for the FIR coefficients was related to the fact that the Impulse Response Function (IRF) of a stable LTI system is exponentially decaying, and moreover, there is a certain level of correlation between the impulse coefficients (smoothness of estimated response).

The regularization methods introduced for FIR modelling are extended to the case of Volterra kernels estimation using the method proposed in [18]. The benefit of regularization in this case with respect to FIR modelling is even more evident given the larger number of parameters usually involved in the Volterra series. Prior information about the Volterra kernels includes the decaying of the kernels as well as the correlation between the coefficients in multiple dimensions. Due to the fact that in case of long measurements and higher order Volterra series, the requested memory needs can be more demanding than the available resources, a memory and computational complexity saving algorithm is provided as well.

In this work, the regularized Volterra kernel estimation technique is combined with a method for transient elimination which plays a key role in this particular benchmark problem because each measurement contains transient. Due to the fact that the measurement length is comparable to the number of parameters, it is necessary to eliminate the undesired effects of the transient as much as possible. The proposed elimination technique uses a special LTI regularization method based on the ideas of an earlier work on nonparametric modelling of LTV systems [19]. It is important to highlight that transient elimination has been not applied yet for the case of Volterra series estimation.

The paper is organized as follows: Section 2 introduces the regularized Volterra kernel estimation method. Section 3 deals with the excessive memory needs of long measurements and large number of parameters. In Section 4 the proposed method for the transient removal is presented. In Section 5 the benchmark problem is formulated and the concrete benchmark results are shown illustrating the efficiency of the combination of the two proposed methods for modelling of the cascaded water tanks system. Early results on the cascaded water tanks benchmark problem can be found in [20]. Finally, the conclusions are provided in Section 6.

2. The nonparametric identification method

2.1. The model structure

It is assumed that the true underlying nonlinear system can be described by the following truncated discrete-time Volterra series [1]:

$$y_{meas}(n) = h_0 + \sum_{m=1}^{M}\left(\sum_{\tau_1=0}^{n_m-1}\cdots\sum_{\tau_m=0}^{n_m-1} h_m(\tau_1,\ldots,\tau_m)\prod_{\tau=\tau_1}^{\tau_m} u(n-\tau)\right) + e(n) \tag{1}$$

where $u(n)$ denotes the input, $y_{meas}(n)$ represents the measured output signal, $e(n)$ is zero mean i.i.d. white noise with finite variance σ^2, $h_m(\tau_1,\ldots,\tau_m)$ is the Volterra kernel of order $m = 1,\ldots,M$, $\tau_i, i = 1,\ldots,m$ denote the lag variables and $n_m - 1$ corresponds to the memory of h_m. The Volterra kernels are considered to be symmetric, which means that [12]:

$$\begin{aligned} &h_n(\tau_1,\tau_2,\ldots,\tau_n) = h_n(\tau_2,\tau_1,\ldots,\tau_n) = \ldots = h_n(\tau_{i_1},\tau_{i_2},\ldots,\tau_{i_n}), i_j \neq i_k \\ &i_1, i_2,\ldots,i_n \in (1,2,\ldots,n), j,k \in (1,2,\ldots,n) \end{aligned} \tag{2}$$

Due to symmetry, it can be easily shown that the number of coefficients to be estimated for a symmetric Volterra kernel of order $m \geqslant 1$ is $n_{h_m} = (\frac{1}{m!})\prod_{i=0}^{m-1}(n_m - i)$. It is also important to clarify the difference between order and degree of the Volterra series with an example: the third degree Volterra series contains the Volterra kernels of order 0, 1, 2 and 3.

2.2. The cost function

Given N input-output measurements from the system to-be-identified, Eq. (1) can be rewritten into a vectorial form as $Y_{meas} = K\theta + E$, where $\theta \in \mathbb{R}^{n_\theta}$, $n_\theta = 1 + \sum_{m=1}^{M} n_{h_m}$, contains vectorised versions of the Volterra kernels h_m, $K \in \mathbb{R}^{N\times n_\theta}$ is the