Automatica 50 (2014) 727–736

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

A variational integrators approach to second order modeling and identification of linear mechanical systems[☆]

Mattia Bruschetta [a], Giorgio Picci [a,1], Alessandro Saccon [b]

[a] Department of Information Engineering, Università di Padova, via Gradenigo 6/B, 35131 Padova, Italy

[b] Department of Mechanical Engineering, Eindhoven University of Technology, The Netherlands

ARTICLE INFO

Article history:
Received 13 April 2011
Received in revised form
12 September 2013
Accepted 20 November 2013
Available online 26 December 2013

Keywords:
Discrete mechanical systems
Variational integrators
Second order models
Subspace identification

ABSTRACT

The theory of variational integration provides a systematic procedure to discretize the equations of motion of a mechanical system, preserving key properties of the continuous time flow. The discrete-time model obtained by variational integration theory inherits structural conditions which in general are not guaranteed under general discretization procedures. We discuss a simple class of variational integrators for linear second order mechanical systems and propose a constrained identification technique which employs simple linear transformation formulas to recover the continuous time parameters of the system from the discrete-time identified model. We test this approach on a simulated eight degrees of freedom system and show that the new procedure leads to an accurate identification of the continuous-time parameters of second-order mechanical systems starting from discrete measured data.

1. Introduction and motivations

The identification of linear second order models of mechanical systems has been the object of intensive research and of several papers in the past decade (De Angelis, Lus, Betti, & Longman, 2002; Lus, De Angelis, Betti, & Longman, 2002, 2003). Of particular interest are systems which can be described by a second order vector model of the following form:

$$M\ddot{q} + D\dot{q} + Kq = f \tag{1.1}$$

where M and K, both symmetric positive definite matrices in $\mathbb{R}^{n\times n}$, have the interpretation of generalized mass (or inertia) and generalized stiffness coefficient matrices respectively, while $D \in \mathbb{R}^{n\times n}$, $D = D^T$ is a linear (viscous) damping coefficient which is at least positive semidefinite. The generalized forces f acting on the system can be expressed as a linear function of a vector of independently assignable generalized input forces u of dimension $m \le n$; namely

$$f = Lu \tag{1.2}$$

where the matrix L, which will be assumed to be known, describes the physical locations at which the input forces u act on the system. Without loss of generality it may be assumed that L is of full column rank; i.e. rank $L = m$.

For simplicity and for mathematical convenience we shall assume that a full set of linear sensors is available to the experimenter; i.e., that all n degrees of freedom are measured via linear sensors. This is obviously equivalent to assume that the measurement equation is $y = q$. The system (1.1) can also be represented in state space form; for example, defining $x := [q,\ \dot{q}]^\top$, one gets

$$\dot{x} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} x + \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix} u \tag{1.3}$$

which should be coupled with the measurement (output) equation $y = [\,I \quad 0\,]\,x$. Note that under our assumptions the system is automatically controllable and observable and hence minimal. This is a necessary condition for parameter identifiability. See Laub and Arnold (1984) for a direct test of controllability/observability of second order models of the type considered in this paper.

Now, in several areas of engineering, such as mechanical or structural engineering, an accurate estimation of the parameters (M, K, D) of the physical *continuous time* model (1.1) is often required. A typical example being the estimation of deformations

☆ This work was supported in part by the National Project *Identification and Adaptive control of Industrial Systems* funded by the Italian Ministry of higher education. The material in this paper was partially presented at the 15th IFAC Symposium on System Identification (SYSID), July 6–8, 2009, St Malo, France. This paper was recommended for publication in revised form by Associate Editor Wolfgang Scherrer under the direction of Editor Torsten Söderström.

E-mail addresses: bruschet@dei.unipd.it (M. Bruschetta), picci@dei.unipd.it (G. Picci), a.saccon@tue.nl (A. Saccon).

1 Tel.: +39 049 8277705; fax: +39 049 8277699.

0005-1098/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.automatica.2013.12.012

at points which are not monitored or the estimation of proper modes of vibration of a mechanical structure.

Mainstream system identification theory deals with discrete-time data and discrete-time models and normally the recovery of the continuous-time parameters involves a conversion step from discrete to continuous time (the so-called *indirect approach*). The problem of reconstructing a continuous-time model from an identified discrete-time model has a long history and has been discussed in several places, see e.g. Söderström (1991) and the reference list in the more recent paper Mahata and Fu (2007). The conversion step from discrete to continuous may sometimes be cause of troubles. It is a commonly experienced fact that for multivariable systems of moderate/large dimension, accurate values of the continuous-time parameters may be hard to recover from the estimated discrete-time system, no matter how accurate the estimates of the latter may be. One reason of this difficulty may be attributed to the ill-conditioning of the discrete-to-continuous conversion, which involves, in the zero-(or first-order)-hold (ZOH) discretizations,[2] inverting the exponential relation $F = \exp Ah$, $G = \int_0^h \exp As\, ds\, B$ to recover the matrices (A, B) of the continuous-time model from estimates (F, G) of an identified discrete time model

$$x_{k+1} = Fx_k + Gf_k, \qquad y_k = Hx_k + Jf_k. \tag{1.4}$$

The default option in the discrete-to-continuous (d2c) routine in MATLAB is ZOH. It is well-known that this operation may turn into an ill-conditioned problem since the recovery of matrix A involves the computation of the logarithm of F which may be a complex matrix or, may be undefined as requiring the inversion of the exponential map in a region of the complex plane where it is not invertible. We would like to point out that the common belief that this problem should be solvable by choosing a high sampling frequency may actually worsen the problem. Consider the trivial case of a scalar discrete F subject to a perturbation δF. The relative error incurred when computing $A + \delta A := \frac{1}{h}\log(F + \delta F)$ is

$$\frac{\delta A}{A} = \frac{1}{\log F}\,\frac{\delta F}{F}$$

a more complicated formula holding in the matrix case, see Dieci and Papini (2000, Formula 2.3). Since for $h \to 0, F \to I$, the condition number of computing $A = \frac{1}{h}\log F$ tends to infinity when $h \to 0$. This means that at high sampling frequency, the effect of unavoidable random errors on the estimates of F (and G) could be largely amplified when computing A by the logarithmic transformation. See Dieci and Papini (2000) and the references therein.

A possible option in the Matlab d2c routine is the so-called *Tustin transform*. Since this discretization scheme has superficial similarities with the approach proposed in this paper and deserves an accurate analysis, we shall postpone a detailed discussion of this option to Section 3.

Now, since the problem we are discussing is a specific parameter estimation problem, the continuous model structure obtained from the discrete-time identified model should be easily transformable into the form (1.1) or (1.3) in that particular basis. In general however, an identified discrete model will just have a generic structure (1.4) with full matrices (F, G, H, J) and need not have any of the structural properties of a mechanical system. In particular, the continuous state-space realization obtained by the inverse of the ZOH or FOH discretizations does not lead to an input–output relation of the special second-order form (1.1). This means that the recovery of the physical parameters M, D, K may in general be ill-defined or impossible. That this is not of purely academic interest is witnessed by the interest in this problem in the recent mechanical engineering literature, see e.g. De Angelis et al. (2002), Lus, De Angelis, and Betti (2003), Lus, De Angelis, Betti, Longman (2003) and the references therein. Ideally, we would like to use discretization schemes which preserve the second order input–output structure of the type (1.1), which is a basic characteristic of linear models of fully observed mechanical systems (Newton law).

In addition, besides the previous difficulties, since the inverse discretization transform is generally non-linear it does introduce bias in the estimates of the continuous-time parameters, even when the estimates of the discrete-time parameters are unbiased and accurate. For this reason a linear (or "approximately linear") discrete-to-continuous conversion would be highly desirable.[3]

1.1. On continuous-time identification

An alternative approach could be to identify the continuous-time parameters directly (the so-called *direct approach*). This could be done in several ways. One may attempt to identify the parameters of the model (1.1) or (1.3) from (discrete) noisy observations directly, by using a continuous-time PEM method. However continuous-time iterative optimization methods are especially sensitive to the choice of good initial estimates of the parameters, particularly when the data sampling frequency may not be suitable for a reasonable numerical approximation of derivatives and gradients. The sensitivity to initial estimates is a serious difficulty especially for multivariable (multi input/multi output) models like the mechanical systems we are dealing with, where good quality initial parameter estimates may be hard to obtain. The problem of getting reasonable initial continuous-time parameter estimates seems indeed to be a non trivial one.

Correlation methods, say by replacing the differentiation operator with the so-called delta operator (Feuer & Goodwin, 1996) or by various approximations of the continuous derivative operator have been proposed (Söderström, Fan, Carlsson, & Bigi, 1997). The approach seems to be advantageous only if the underlying continuous system is scalar autoregressive. The accuracy of the approach depends on the particular approximation being used and does not seem to be easy to assess, especially when the method should be applied to multivariable continuous-time models of moderate or high dimension. Other continuous-time identification algorithms eventually end up to rely on logarithmic transforms, like inverting the relation $z = \exp\{sh\}$ which turns out to be equivalent to the logarithmic d2c transformation. For the reasons given above, these methods are not always reliable. There is also a quite popular approach based on filtering the continuous-time data by a family of test functions (Heuberger, de Hoog, van den Hof, & Wahlberg, 2003; Ohta & Kawai, 2004), which may or may not be orthonormal. This approach needs extensive numerical integration for computing the inner products over a long period of time, since in order to reach a reasonable accuracy the computation of many inner products of the measured signal with a large number of test functions is needed. In a sense each inner product plays eventually the role of a single discrete-time sample value of the signal.

Unfortunately according to the current literature on continuous time identification, see e.g. Garnier and Wang (2008), Sinha and Rao (1991) and the references therein, the existing

[2] The ZOH sampler transforms continuous-time into discrete-time by synchronously sampling the output of the continuous system once the input signal is approximated by a piecewise constant function on each sampling interval. There are more refined schemes, such as the first-order-hold (FOH) which assumes instead the input to be piecewise linear. The discussion which follows applies also to FOH, modulo notational complications which we choose to avoid.

[3] One may argue that the Euler discretization is a well known instance of linear conversion map but unfortunately the Euler discretization is too primitive to be of use in most situations.