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Analytically optimal parameters of dynamic vibration absorber with negative stiffness

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ABSTRACT

In this paper the optimal parameters of a dynamic vibration absorber (DVA) with negative stiffness is analytically studied. The analytical solution is obtained by Laplace transform method when the primary system is subjected to harmonic excitation. The research shows there are still two fixed points independent of the absorber damping in the amplitude-frequency curve of the primary system when the system contains negative stiffness. Then the optimum frequency ratio and optimum damping ratio are respectively obtained based on the fixed-point theory. A new strategy is proposed to obtain the optimum negative stiffness ratio and make the system remain stable at the same time. At last the control performance of the presented DVA is compared with those of three existing typical DVAs, which were presented by Den Hartog, Ren and Sims respectively. The comparison results in harmonic and random excitation show that the presented DVA in this paper could not only reduce the peak value of the amplitude-frequency curve of the primary system significantly, but also broaden the efficient frequency range of vibration mitigation.

1. Introduction

Vibration control has been important for several decades and many efficient devices have been presented. One of the common devices for vibration control is dynamic vibration absorber (DVA), which is widely used for its properties such as efficiency, reliability, and low cost. Researches on DVA have been developed for more than 100 years since the first DVA without damping was invented by Frahm [1] in 1909. In 1928, Den Hartog and Ormondroyd [2] found that a DVA with damping element could suppress the amplitude of the primary system in a broader frequency range, which had been recognized as the typical Voigt type DVA. Den Hartog and Ormondroyd also found that the amplitude-frequency curve of the damped DVA would pass through two fixed points independent of the absorber damping, and they proposed an optimization criterion to design the Voigt type DVA. Hahnkamm [3] derived the optimum natural frequency ratio in 1932 according to this criterion. Later, the optimum damping ratio was obtained by Brock [4]. In 1956, Den Hartog gave a detailed introduction on this optimization criterion in his famous monograph [5], and he called this method as the fixed-point theory. In 2002, Nishihara and Asami [6] derived the exact series solutions for the optimum frequency and damping ratios of the Voigt type DVA, and compared the control performance with that by Den Hartog. In 2007, Sims [7] introduced a new

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analytical solution based on the criterion of minimizing either the positive real part or negative real part of the frequency response function, so as to efficiently control the chattering phenomenon in machining tool. In 2001, Ren [8] presented a DVA where the damping element was not connected to the primary system, but to the earth or the base structure. The result indicated it could present better control performance than Voigt type DVA under the same parameters condition. In 2005, Liu [9] got the same result using another method. However it was hard to get the analytical optimal solution when the damping of the primary system could not be ignored, so that some numerical methods were proposed to get the optimal solution [10–12]. In addition to the above passive DVAs, Shen et al. [13,14] studied the approximately analytical solutions for four types of semi-active DVAs, and the effect of the time delay on the control performance was also studied.

The positive stiffness means that the deformation is in the same direction as the applied exterior force, which is commonly encountered in practical engineering. On the contrary, the negative stiffness means the relationship between the exterior force and the displacement in deformed objects is reverse. The researches about the properties and stability condition of the negative stiffness system had been reported in references [15–21]. There are some generation mechanisms for negative stiffness, where the pre-compressed member or inverted pendulum are typical devices. When the springs with positive and negative stiffness are put in parallel and connected to different masses, the system may show nonlinear characteristic near the equilibrium position, such as instability or bifurcation. The load-bearing capacity of the system with negative stiffness is better than that of the system with only positive stiffness, and at the same time the natural frequency of the system will be reduced. The system with negative stiffness may have a better vibration control performance if it remains stable. Thus, the introduction of the negative stiffness to the vibration control system is necessary and meaningful. Platus et al. [22] produced negative stiffness around equilibrium using the buckling of beams under axial load, and they got an isolation system by combining it with a linear positive spring. Trimboli [23] proposed the application of the negative stiffness in the mechanical vibration isolator, where the negative stiffness and positive stiffness were arranged in parallel. Park [24] studied the active control vibration isolator with negative stiffness, and the basic system characteristics were experimentally verified. Mizuno et al. [25–28] studied an active vibration isolation system combining zero-power magnetic suspension analytically and experimentally. They found that a zero-power system behaved like negative stiffness system, and could generate infinite stiffness if it was connected with a normal spring in series. Then they proposed a new vibration isolation system, where the negative stiffness was realized by active control technique [29]. In 2013, Acar et al. [30] studied an adaptively passive DVA with a negative stiffness mechanism analytically and experimentally, and found that it could suppress the system amplitude by appropriately adjusting the parameters. Yang et al. [31] also investigated a nonlinear vibration isolator with a negative stiffness mechanism. Many scholars had studied the application of the negative stiffness in vibration isolation system for its advantages, but little work has been done about the optimal parameters of the DVA with negative stiffness until now.

In this paper the effect of the negative stiffness on the amplitude of the primary system is studied by introducing the negative stiffness which is connected the DVA with the earth. Section 2 presents the optimal parameters based on the fixed-point theory. In Section 3, the control performance of the presented DVA in this paper is compared with those of the DVAs by Den Hartog, Ren and Sims when subjected to both sinusoidal and random excitations. The results verify the DVA in this paper has more significant control performance. At last the conclusions are made in Section 4.

2. Analytical investigation on DVA with negative stiffness

The typical Voigt type DVA proposed by Den Hartog is shown in Fig. 1(a), and the DVA proposed by Ren is shown in Fig. 1(b). By adding the negative stiffness directly connecting the mass of DVA and the earth in the model by Ren, one could get the model with negative stiffness shown in Fig. 1(c).

Obviously, the model in Fig. 1(b) is a special case of Fig. 1(c), if the coefficient of the negative stiffness is set to zero. According to Newton's second law, the motion equation of the system with negative stiffness can be established as

$$\begin{cases} m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = F\cos(\omega t) \\ m_2\ddot{x}_2 + c\dot{x}_2 + (k + k_2)x_2 - k_2x_1 = 0 \end{cases}, \tag{1}$$

where m_1, m_2, k_1 and k_2 are the masses, linear stiffness coefficients of the primary system and the DVA respectively. c is the damping coefficient of the absorber, and k is the negative stiffness coefficient. x_1 and x_2 are the displacements of the primary system and the DVA respectively. F and ω are the amplitude and frequency of the force excitation.

Using the following parametric transformation

$$\mu = \frac{m_2}{m_1},\ \omega_1 = \sqrt{\frac{k_1}{m_1}},\ \omega_2 = \sqrt{\frac{k_2}{m_2}},\ \xi = \frac{c}{2m_2\omega_2},\ \alpha = \frac{k}{k_2},\ f = \frac{F}{m_1},$$

Eq. (1) becomes

$$\begin{cases} \ddot{x}_1 + (\omega_1^2 + \mu\omega_2^2)x_1 - \mu\omega_2^2x_2 = f\cos(\omega t) \\ \ddot{x}_2 + 2\omega_2\xi\dot{x}_2 + (\alpha + 1)\omega_2^2x_2 - \omega_2^2x_1 = 0 \end{cases}. \tag{2}$$