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Accurate modal superposition method for harmonic frequency response sensitivity of non-classically damped systems with lower-higher-modal truncation

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ABSTRACT

Frequency response and their sensitivities analysis are of fundamental importance. Due to the fact that the mode truncation errors of frequency response functions (FRFs) are introduced for two times, the errors of frequency response sensitivities may be larger than other dynamic analysis. Many modal correction approaches (such as modal acceleration methods, dynamic correction methods, force derivation methods and accurate modal superposition methods) have been presented to eliminate the modal-truncation error. However, these approaches are just suitable to the case of un-damped or classically damped systems. The state-space equation based approaches can extend these approaches to non-classically damped systems, but it may be not only computationally expensive, but also lack physical insight provided by the superposition of the complex modes of the equation of motion with original space. This paper is aimed at dealing with the lower-higher-modal truncation problem of harmonic frequency response sensitivity of non-classically damped systems. Based on the Neumann expansion and the frequency shifting technique, the contribution of the truncated lower and higher modes to the harmonic frequency response sensitivity is explicitly expressed only by the available middle modes and system matrices. An extended hybrid expansion method (EHEM) is then proposed by expressing harmonic frequency response sensitivity as the explicit expression of the middle modes and system matrices. The EHEM maintains original-space without having to involve the state-space equation of motion such that it is efficient in computational effort and storage capacity. Finally, a rail specimen is used to illustrate the effectiveness of the proposed method.

1. Introduction

Harmonic frequency response sensitivity analysis deals with the calculations of the rate of frequency response from changes in design parameters of structural and mechanical systems subjected to harmonic loading. Frequency response sensitivity of structural systems is of fundamental importance and plays a very important role in many areas such as model updating [1,2], vibration and noise control [3,4], structural damage detection [5,6], system identification [7–9] and dynamic

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optimization [10,11] and many other applications.

Generally speaking, two kinds of method are usually used to calculate the frequency response sensitivity, i.e., direct frequency response method (DFRM) and modal superposition method. The DFRM calculate the frequency response sensitivity in a manner similar to a static-force problem at each excitation frequency. It is an exact method but it requires the decomposition of the dynamic stiffness matrix for each excitation frequency. It is time-consuming when the number of degree of freedom (DOF) and excitation frequency is large. The modal superposition method calculates the frequency response sensitivity by expressing it as the summation of the contributions of all the modes. In order to obtain the accurate results, the method requires all the modes to produce an exact result. However, usually it is difficult, or even impossible, to obtain all the modes of large-scale structures. Therefore, the mode displacement method (MDM), which only uses the lower modes to approximately calculate dynamic response and the sensitivities, is often used in engineering applications. As a result, the modal truncation error is generally introduced and the quality of the calculated results may be adversely affected.

To correct the modal truncation error, a large number of highly effective methods including modal acceleration methods [12–16], dynamic correction methods [17–19], force derivation methods [20,21] and accurate modal superposition methods [22–28] have been proposed. However, those methods are only restricted to the case of un-damped or classically damped systems. A classically damped system means that energy is almost uniformly dissipated throughout the mechanical system and the system equation can be decoupled using normal modes. Otherwise, it is a non-classically damped system. Due to the two widely accepted criteria (Hasselman criterion and diagonal matrix criterion) are no longer applicable, these proportional approximation methods above-mentioned are not suitable for approximation the frequency response of the non-classical damping system. In order to extend modal correction approaches used in un-damped or classically damped systems to the non-classically damped systems, the state-space equation based approaches [26,29] are proposed to approximation the dynamic response. The state-space equation based approaches are exact in nature. However, the approaches are not only often require heavy computational cost for practical engineering application since the size of system matrices of state-space equations is double, but also lack the physical insight provided by superposition of the complex modes of the equation of motion in the original space. To save computational resource, Li [30,31] present some methods which maintain original-space without having to involve the state-space equation of motion to correct the modal truncation error. The methods express the contribution of the truncated higher modes as expression comprised of the lower modes and system matrices based on the Neumann expansion.

According to the frequency range of truncated modes, there are middle-higher modal truncation and lower-higher-modal truncation. In the first case, the lower modes are retained, while the middle modes are available in the second case. In the dynamic analysis of coupled acoustic-structural systems [23], the available modes are located in the middle frequency range. In addition, general commercial finite element software provides the modal analysis located in the middle frequency range. It means the lower-higher-modal truncation error is introduced. Unfortunately, the correction method which is applied to the middle-higher modal truncation error cannot be directly used for the lower-higher-modal truncation problem. In the past decades, some correction methods [23,27,32] are presented to correct the lower-higher-modal truncation error.

This paper is aimed to calculate accurately the harmonic frequency response sensitivity of non-classically damped systems in terms of the modes in the middle frequency range. Based on the Neumann expansion and frequency shifting technique, an extended hybrid expansion method (EHEM) is proposed in this study to deal with the lower-higher-modal truncation problem. Since frequency response function (FRF) is repeated twice in the harmonic frequency response sensitivity analysis, the lower-higher-modal truncation error is introduced for two times using the complex modal superposition. As a result, the calculation error of sensitivity analysis is larger than other dynamic analysis. In the presented EHEM, the error of the truncated lower and higher modes (unavailable) is corrected by a convergent and explicit power-series expansion which only comprises by the middle modes (available) and system matrices. A rail specimen will be used to illustrate the effectiveness of the derived results. It will be shown that the modal truncation error of the harmonic frequency response sensitivities of non-classically damped systems can be significantly reduced by considering the proposed method.

2. Harmonic frequency response sensitivity analysis of non-classically damped systems

The motion equations for a multi-degree-of-freedom (MDOF) non-classically damped system subjected to external dynamic forces can be expressed as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}\in\mathbb{R}^{N\times N}$, $\mathbf{C}\in\mathbb{R}^{N\times N}$ and $\mathbf{K}\in\mathbb{R}^{N\times N}$ are, respectively, the mass, viscous damping and stiffness matrices (only consider symmetric system matrices here), $\mathbf{q}(t)$ and $\mathbf{f}(t)$ are the displacement and force vectors, respectively. In general, $\mathbf{C}$ and $\mathbf{K}$ are nonnegative definite symmetric matrices and $\mathbf{M}$ is a positive definite symmetric matrix. When the damped system is subjected to harmonic excitation, the force vector $\mathbf{f}(t)$ in Eq. (1) takes the form, $\mathbf{f}(t) = \mathbf{F}_h\exp(\mathrm{i}\omega t)$, and the resulting steady-state response $\mathbf{q}(t)$ is also harmonic motion which takes the form of $\mathbf{q}(t) = \mathbf{X}(\mathrm{i}\omega)\exp(\mathrm{i}\omega t)$ where $\mathbf{X}(\mathrm{i}\omega)$ is the complex frequency response vector. Thus, in the frequency domain, the governing dynamic responses of the damped system subjected to harmonic excitation can be rewritten in the following form: