Mechanical Systems and Signal Processing 85 (2017) 487–511

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Uncertain dynamic analysis for rigid-flexible mechanisms with random geometry and material properties

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ARTICLE INFO

Article history:
Received 20 May 2016
Received in revised form
1 August 2016
Accepted 28 August 2016

Keywords:
Rigid-flexible multibody systems
Absolute node coordinate formula (ANCF)
Random uncertainty
Polynomial Chaos (PC) expansion

ABSTRACT

This paper proposes an uncertain modelling and computational method to analyze dynamic responses of rigid-flexible multibody systems (or mechanisms) with random geometry and material properties. Firstly, the deterministic model for the rigid-flexible multibody system is built with the absolute node coordinate formula (ANCF), in which the flexible parts are modeled by using ANCF elements, while the rigid parts are described by ANCF reference nodes (ANCF-RNs). Secondly, uncertainty for the geometry of rigid parts is expressed as uniform random variables, while the uncertainty for the material properties of flexible parts is modeled as a continuous random field, which is further discretized to Gaussian random variables using a series expansion method. Finally, a non-intrusive numerical method is developed to solve the dynamic equations of systems involving both types of random variables, which systematically integrates the deterministic generalized-$\boldsymbol{\alpha}$ solver with Latin Hypercube sampling (LHS) and Polynomial Chaos (PC) expansion. The benchmark slider-crank mechanism is used as a numerical example to demonstrate the characteristics of the proposed method.

1. Introduction

As one of the most important systems in mechanical engineering, almost all the multibody dynamic mechanical systems (or mechanisms) involve various uncertain factors, which may influence the performance of a system especially for the high-speed dynamic systems. For instance, the geometry size of a component in the mechanism has a tolerance to facilitate manufacturing process; the fabrication of different kinds of raw material may lead to the inhomogeneous distribution of the material, which will further lead to the variation of material properties, such as the Young's modulus, Poisson's ratio, and material density. To improve the computational accuracy of dynamic analysis of the mechanism, it is necessary to investigate their dynamic responses by considering these unavoidable uncertain factors.

The dynamic study of the mechanisms under deterministic conditions has been developed from traditional rigid multibody systems to flexible multibody systems and rigid-flexible multibody systems. The modelling of rigid multibody systems has been well studied [1], and some commercial software has also been widely used. On the other hand, the study of flexible multibody systems and rigid-flexible multibody systems has been attracting more and more attention over the past two decades. When the flexible components are involved in multibody systems, the deformation of these flexible

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http://dx.doi.org/10.1016/j.ymssp.2016.08.040
0888-3270/

components has to be considered. Since flexible components in multibody systems often experience large rotation and deformation, the traditional finite element methods based on the small rotation and deformation may give an improper solution [2]. However, the Absolute Nodal Coordinate Formulation (ANCF) [3] shows good capability for solving flexible multibody problems with large rotation and deformation. ANCF defines elemental coordinates as the absolute displacements and global slopes, which forces the mass matrix of the system equations to remain constant and the centrifugal and Coriolis forces identically equal to zero [4,5]. As a non-incremental finite element method, the ANCF has been considered as an effective approach for analysis of flexible multibody dynamics [6]. There have been some applications with the flexible multibody systems solved by using the ANCF, such as the flexible multibody systems with viscoelastic materials [7], the clearance and lubricated joints problems [8], and large deformation problems [9–12].

To solve the rigid-flexible multibody systems, Tian et al. [13] combined ANCF with Natural Coordinate Formulation (NCF) to build dynamic models of rigid-flexible multibody systems, in which the flexible parts were modeled by using ANCF elements and the rigid parts were described by the NCF. Liu et al. [14] used the ANCF-NCF method to compute the dynamic response of a large scale rigid-flexible multibody system composed of composite laminated plates. Similar to the concept of NCF, Shabana [15] employed the ANCF reference nodes (ANCF-RNs) to describe the rigid body. As a result, the complicated rigid-flexible multibody systems could be modeled by using the ANCF-based method, in which the flexible parts were built by the ANCF elements while the rigid parts were described by the ANCF-RNs. More applications about the ANCF-based method on the rigid-flexible multibody systems can be found in [16].

To solve the multibody systems containing uncertain parameters, not only the aforementioned deterministic methods but also some uncertain analysis methods have to be used. Based on the different characteristics of uncertain parameters, the uncertain analysis methods can be divided into two categories, which are the probabilistic methods to solve the random parameters and the non-probabilistic methods to solve other non-probabilistic parameters [17,18]. Non-probabilistic methods [19–25] are mainly used as a complimentary of probabilistic methods when the probabilistic information of uncertain parameters may not be obtained. There also have been some references focusing on the hybrid uncertain methods [26,27]. This paper is focused on the probabilistic methods by assuming the characteristics of random parameters are known.

In a rigid-flexible multibody system, the uncertain parameters of the rigid parts may be directly described by random variables, e.g. the mass, mass center, mass moment of inertia, and geometry size. Specifically, the geometry size is certainly subject to uncertainty for the tolerance of manufacture, and it will lead to the uncertainty of other parameters, e.g. mass center, mass, mass moment of inertia, gravity force, and constraint conditions. On the other hand, the inhomogeneous distribution of material in space may result in uncertainty of material properties (e.g. Young's modulus and Poisson's ratio), which may continuously vary. Therefore, the uncertainties of material properties should be described as random fields. The random field has been widely used in structure analysis [28–30], but it is rarely used in the dynamic computation of multibody systems.

The continuous random field is defined as an indexed set of random variables, and the index belongs to some continuous uncountable set. Since the uncountable index set is not convenient to implement in the computation, the random field needs to be discretized into a set of countable random variables. Ghanem and Spanos [30] has stated that a continuous random field can be approximately represented by a set of countable random variables. The most widely used discretization methods for the random field are series expansion methods, which aim at expanding any realization of the original random field over a complete set of deterministic functions [31]. After the series is obtained, the discretization is implemented by truncating the series after a finite number of terms. Karhunen-Loeve (K-L) expansion [30], orthogonal series expansion (OSE) [32], and expansion optimal linear estimation (EOLE) method [33] all belong to the series expansion methods. The K-L expansion method has the highest discretization accuracy, but only few covariance functions (e.g. the exponential autocorrelation function) have a closed-form solution for K-L expansion. Numerical methods have to be used to realize the K-L expansion, which can be found in references [29,30]. However, the orthogonal basis of the expansion obtained by most of the numerical methods is no more optimal [31]. The EOLE can be considered as a special case of the Nystrom method that is a type of numerical method to implement the K-L expansion [29], and it is based on an optimal basis. The paper [31] gave a comparison of the discretization methods and indicated that the EOLE could be used in more general cases with high accuracy.

After the discretization, the uncertain material properties of the flexible parts have been transformed to several random variables, the same as other random parameters in the rigid parts. At this stage, the key is to obtain the uncertain characteristics of response (output) from the input random variables, which is the about uncertainty propagation of the random variables. The statistical method [34] is the first strategy to handle this problem, which collects a large number of samples of the random variables according to their probability distribution and then estimates the mean, variance, and even the probability distribution function of the output directly. The Monte Carlo method [35] is one important statistical method, but its accuracy depends on the sampling size, in accordance with the weak law of large numbers. Therefore, to get sufficient accuracy, it usually requires thousands of samples, which is quite expensive for the dynamic computation of rigid-flexible multibody systems. As a result, the Monte Carlo method is often used as the reference of other methods. The samples of Monte Carlo method are produced randomly, some other sampling strategies can be used to improve the convergence ratio of Monte Carlo method, such as the Latin Hypercube sampling (LHS) [36], Orthogonal Sampling [37], low discrepancy sampling [38] and so on [39]. This paper will use the LHS method to improve the convergence ratio of statistical method.

Non-statistical methods [34] can also be used to handle the random variables, e.g. the perturbation method, Neumann