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Interpretation and generalization of complexity pursuit for the blind separation of modal contributions

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ABSTRACT

Complexity pursuit (CP) has recently been proposed as an elegant and simple solution to blindly (i.e. without measuring the inputs) separate the modal contributions in the vibration responses of a structure. This potentially finds considerable interest in operational modal analysis and related applications. This paper analyses the theoretical ins and outs of the method. It also revises its physical interpretation in the modal analysis context. CP is found to separate components which are the least dispersive (i.e. invariant under linear filtering), a property that well characterizes the modal responses of lightly damped systems. However, it is also found to suffer from the same limitations as other blind source separation methods used in the state-of-the-art, namely the difficulty to separate strongly coupled modes and to identify complex mode shapes. Finally a generalization of CP is proposed which intends to widen its applicability. Interestingly, the generalized CreP happens to include the well-known SOBI algorithm as a particular case.

1. Introduction

Complexity Pursuit (CP), a new blind source separation (BSS) technique, was recently introduced in [1–3] and demonstrated to decompose the vibration responses of a structure into individual modal contributions. Such a technique is of particular interest within the context of operational modal analysis (OMA) due to its ability to blindly (i.e. without measuring the inputs) decouple a multiple-degree-of-freedom system into a set of single-degree-of-freedom components, as demonstrated in precursory works [5–8,10–13] and in later developments [14]. In particular, it can greatly simplify the subsequent identification task required for extracting the modal information from the system responses: the global modal parameters (natural frequencies and damping ratios) can easily be identified by using single-degree-of-freedom methods and the mode shape estimated from the inverse of the separation matrix. The BSS technique of [1] is an adaptation of the complexity pursuit (CP) principle initially formulated in [4] in a statistical learning context (note that [15] independently formulated a BSS method based on a similar idea). Basically, it consists in finding a modal filter intended to extracting an individual modal contribution by minimizing the energy ratio between two filtered versions of the output signals. Namely, using the notations of [1], let the $n \times 1$ column vector

$$\mathbf{x}(t) = \mathbf{\Phi}\mathbf{q}(t) = \sum_{i=1}^{n} \boldsymbol{\varphi}_i q_i(t) \tag{1}$$

Abbreviations: AMUSE, Algorithm for multiple unknown signals extraction (algorithm); BSS, Blind source separation; CP, Complexity Pursuit (algorithm); DOF, Degrees of freedom; GCP, Generalized Complexity Pursuit (algorithm); OMA, Operational modal analysis; SOBI, Second order blind identification (algorithm)

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denotes the observed vibration responses of a structure at time t produced by a mixture of modal coordinates $q_i(t)$ weighted by mode shapes $\boldsymbol{\varphi}_i$, $i = 1, \ldots, n$ (columns of the $n \times n$ modal matrix $\boldsymbol{\Phi}$). The objective is to find a modal filter $\mathbf{w}_i$ (a $1 \times n$ raw vector) such that

$$y_i(t) = \mathbf{w}_i \mathbf{x}(t) \tag{2}$$

returns an estimate of $q_i(t)$ up to a scaling factor, from which global modal parameters can be subsequently recovered, on the one hand. One the other hand, the inverse of matrix $\mathbf{W}$ made of the rows $\mathbf{w}_i$, $i = 1, \ldots, n$ is an estimate of the modal matrix $\boldsymbol{\Phi}$ that contains information on the mode shapes. The principle of CP is to estimate $\mathbf{w}_i$ such as to minimize the ratio

$$\rho_i = \frac{\langle |y_i^{(1)}(t)|^2 \rangle}{\langle |y_i^{(2)}(t)|^2 \rangle} = \frac{\mathbf{w}_i \langle \mathbf{x}^{(1)}(t)\mathbf{x}^{(1)}(t)^T \rangle \mathbf{w}_i^T}{\mathbf{w}_i \langle \mathbf{x}^{(2)}(t)\mathbf{x}^{(2)}(t)^T \rangle \mathbf{w}_i^T} \tag{3}$$

where $\langle \cdots \rangle$ stands for the time average operator and superscript $k = 1, 2$ symbolically represents two different filtered versions of the same signals. In its original formulation, $\mathbf{x}^{(1)}(t)$ (resp. $\mathbf{x}^{(2)}(t)$) is the residual between the actual signals $\mathbf{x}(t)$and a long-term predictor $\bar{\mathbf{x}}^{(1)}(t)$ (resp. short –term predictor $\bar{\mathbf{x}}^{(2)}(t)$)

$$\begin{cases} \mathbf{x}^{(k)}(t) = \bar{\mathbf{x}}^{(k)}(t) - \mathbf{x}(t) \\ \bar{\mathbf{x}}^{(k)}(t) = \lambda_k \bar{\mathbf{x}}^{(k)}(t-1) + (1-\lambda_k)\mathbf{x}(t-1) \end{cases}, \\ 0 \le \lambda_1 < \lambda_2 \le 1, \quad k = 1, 2 \tag{4}$$

[1–3] used $\lambda_1 = 2^{-1/900000}$ and $\lambda_2 = 0.5$ by default.

As stated in [1], the rationale beyond criterion (3) is to seek a separation vector $\mathbf{w}_i$ that yields the "least complexity and thus approaches the (simplest) source signal, where the complexity is robustly measured by temporal predictability". Using the authors' words, CP is "computational efficient, user-friendly, and automatic, requiring little expertise interactions for implementations". Indeed, the minimization of criterion (3) with respect to $\mathbf{w}_i$ simply amounts to computing the generalized eigenvectors of the cross-correlation matrices $\langle \mathbf{x}^{(1)}(t)\mathbf{x}^{(1)}(t)^T \rangle$ and $\langle \mathbf{x}^{(2)}(t)\mathbf{x}^{(2)}(t)^T \rangle$ (see Section 3.1).

A first objective of this paper is to provide the theoretical foundation and physical interpretation of the CP that are lacking in [1]. In view of its potential importance to OMA, it is imperative to establish at the onset whether it is capable of separating modal contributions in the general scenario and, if not, to delineate its limits. In particular, the question arises as whether it can resolve highly coupled modes (e.g. closely spaced frequencies and/or strongly damped modes) and complex mode shapes, two configurations which are still challenging to cutting-edge BSS techniques [14]. Second, it is also compulsory to compare its performance against SOBI [17], a state-of-the-art BSS technique used in OMA which has been standing as a point of reference since a few years [7,8,10,14].

A second objective of the paper is to propose a generalization of CP that is shown to apply more widely and provides perspective for the proposal of new BSS algorithms.

The main results of this paper are summarized hereafter:

1) The solutions of CP, as formulated by Eqs. (2)–(4), are pure sines. Strictly speaking, this generally precludes the *exact* recovery of vibration modes as soon as damping is present in the system.
2) However, very good separation of lightly damped modes is to be expected provided the built-in filters of CP are smooth enough to be considered as approximately constant across the mode bandwidths.
3) The original formulation of CP in terms of short and long-term predictors (Eq. (4)) can be generalized to the consideration of any type of filters, provided they are smooth enough in the sense of point (2). This makes obsolete the interpretation of CP as seeking for the least complex components that are "maximally predictable".
4) The physical interpretation of the CP is to seek vibration components that remain invariant under arbitrary (linear) filtering. In terms of waveforms, these are components which are as least dispersive as possible, that is nearly invariant under linear filtering. Non-dispersion is an intrinsic property of pure sines, yet it can be approached remarkably well by lightly damped modes whose modal coordinates resemble slowly modulated sinusoids. Least complexity in CP is therefore to be measured by "dispersion" rather than by "predictability".
5) CP is generally unable to recover complex mode shapes (e.g. in the case of non-proportional damping) since the generalized eigenvectors of square real symmetric matrices $\langle \mathbf{x}^{(1)}(t)\mathbf{x}^{(1)}(t)^T \rangle$ and $\langle \mathbf{x}^{(2)}(t)\mathbf{x}^{(2)}(t)^T \rangle$ are real-valued, unless specific pre-processing is used as suggested in [10].
6) CP presents a strong analogy with AMUSE, the two time-lag version of SOBI [18]. Some particular choices of the built-in filters can make it identical to AMUSE.
7) Simulations show that CP is not superior to SOBI in the general case and that it suffers from the same difficulty to separate strongly coupled modes and complex mode shapes.
8) A generalization of the original CP algorithm is proposed that involves an arbitrary number of filters. This involves an approximate joint diagonalization of a set of cross-correlation matrices which is likely to improve the performance of the plain-vanilla CP method. The generalization includes SOBI as a particular case.
9) Several sets of filters are tested on simulated and real data in order to demonstrate how to optimize the separation of vibration components. One advantage of the generalized CP is to provide a versatile algorithm that is intended to shortcut this step.