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Efficient sensor placement for state estimation in structural dynamics

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ABSTRACT

This paper derives a computationally efficient algorithm to determine optimal sequential sensor placement for state estimation in linear structural systems subjected to unmeasured excitations and noise contaminated measurements. The proposed algorithm is developed within the context of the Kalman filter and it minimizes the variance of the state estimate among all possible sequential sensor locations. The paper investigates the effects of measurement type, covariance matrix partition selection, spatial correlation of excitation and model selection on optimal sensor placement. The paper shows that the sequential approach reaches the optimal sensor placement as the number of sensor increases.

1. Introduction

State estimation is a branch of control theory that deals with estimating hidden variables (the state) of a dynamical system based on a model and noise contaminated response measurements. This paper deals with state estimation in linear structural dynamics and in particular with the problem of determining optimal sensor locations such that the error in the estimated state or functions of the state is minimized. In this paper, the problem of optimal sensor placement will be defined as follows: given a linear model and covariance matrices for unmeasured loads and measurement noise, determine the particular sensor layout that minimizes the trace of the state error covariance of a user defined linear function of the state. A particular function of interest is the one that maps displacements at all degrees of freedom with the strains at points of interest. This function is significant in many applications, including fatigue monitoring of structures using global response measurements [1,2].

One of the first documented efforts to apply state estimation in the context of structural dynamics was presented by Carmichael [3]. Subsequently, Waller and Schmidt [4,5] applied state estimation for monitoring stresses in critical and unaccessible locations in rotating machinery. They implemented the idea of a modal observer as a means to reduce the computational burden when working with large FE models. For systems in which the response can be spanned by a relatively small modal subspace, the estimator (or observer) can be implemented on a reduced order model that only contains the modes of interest. Wilde and Kozakiewicz [6] applied the Kalman filter to estimate the vibration of simple cylindrical cantilever structures partially submerged in water and subject to the interacting motion of the waves. In addition, the Kalman filter was also used to determine hydrodynamic force coefficients under certain steady state conditions. Hernandez and Bernal [7] proposed a robust observer to estimate the state of structural system with model uncertainties in mass, stiffness and (or) damping. More recently various researchers have proposed the use of state estimation as a means to track stress and strain fields in operational structures subjected to unmeasured excitations, the main objective being monitoring fatigue damage [1,2,8–11].

The most common algorithm for state estimation is the Kalman filter [12–14]. In the Kalman filtering, the state of the system is

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estimated recursively at discrete time steps assuming a model of the system and the covariance matrices of the excitations and measurement noise. In the basic Kalman filter formulation it is assumed that the excitations and measurement noise are Gaussian white noise sequences. Within the context of Kalman filtering, optimal sensor placement is achieved by selecting the state-to-output matrix such that the variance of the user-selected state function is minimized. Sometimes the trace of the state error covariance matrix is used as a global measure of optimality, however this is not always adequate if only a subset of states are of interest. For $m > 1$ sensors, the problem of optimal sensor placement becomes a combinatorial problem, i.e. selecting among all possible combinations of placing m sensors in n possible locations. In most practical applications $n \gg m$, which makes the problem of optimal sensor placement computationally challenging.

This paper derives an efficient algorithm to determine sequential and near-optimal sensor placement which includes the possibility of spatio-temporal correlation in the excitations and measurement noise. The variance minimization approach has been employed by other researchers to determine optimal sensor placement in parameter identification and system identification of linear structural systems [16–18].

Most recently the problem of optimal sensor placement for state estimation and joint state-input estimation has been treated by Weickgenannt at al. [20], van den Linder et al. [19] and Zhang and Wang [21]. In [20] an optimal sensor location strategy was developed for an active shell structure. The approach was based on a dual-objective optimization, namely minimizing the number of sensors and a measure related to observability. The optimization was solved using a multi-objective simulated annealing algorithm. In [19], three approaches for optimal sensor placement were compared, namely: minimizing the static displacement estimation error, minimizing the inverse of the observability, and minimizing the dynamic displacement estimation error. All three approaches were found to provide similar error reduction, however their implementation cost was found to vary significantly, with the static approach being the least expensive by orders of magnitude. Each of the approaches was implemented and applied to a 2D model of the long-span New Carquinez Bridge in California. In [21] a sequential optimal sensor placement algorithm based on variance minimization was proposed. Finally, the use of observability as a criteria for optimal sensor placement for state estimation has been proposed and effectively implemented by other authors [19]. We compare the observability approach with the minimum variance approach and show under what conditions it may be more accurate.

The remainder of the paper is organized as follows, the first section presents the necessary theoretical background related to the systems of interest. This is followed by sections on observability, state estimation and Kalman filtering. The paper continues with a section describing optimal sensor placement in the context of the Kalman filter. This is followed by sections presenting numerical simulations that compare the effects of measurement types, covariance partition, spatio-temporal correlation of the input and observability versus minimum variance approach for optimal sensor placement. The paper ends with some final remarks and illustrative examples regarding the optimality and efficiency of the proposed sequential approach and conclusions.

2. Models of interest

This paper is restricted to systems whose dynamic response can be accurately described by the following matrix ordinary differential equation

$$\mathbf{M}\ddot{q}(t) + \mathbf{C}_d\dot{q}(t) + \mathbf{K}q(t) = \mathbf{b}_1 u(t) + \mathbf{b}_2 w(t) \tag{1}$$

where $\mathbf{M} = \mathbf{M}^T > 0 \in R^{N\times N}$ is the mass matrix, $\mathbf{C}_d = \mathbf{C}_d^T > 0 \in R^{N\times N}$ is the damping matrix and $\mathbf{K} = \mathbf{K}^T > 0 \in R^{N\times N}$ is the stiffness matrix. The vector $q(t) \in R^{N\times 1}$ is the displacement vector of the N degrees of freedom, $\mathbf{b}_1 \in R^{N\times r}$ defines the spatial distribution of the known excitation $u(t) \in R^{r\times 1}$ and $\mathbf{b}_2 \in R^{N\times p}$ defines the spatial distribution of the unmeasured excitation $w(t) \in R^{p\times 1}$. By defining the state vector as $x(t) = [q^T(t) \quad \dot{q}^T(t)]^T \in \mathbb{R}^{2N\times 1}$, Eq. (1) can be written in first order state-space form as

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}_1 u(t) + \mathbf{B}_2 w(t) \tag{2}$$

where the matrices $\mathbf{A}$, $\mathbf{B}_1$ and $\mathbf{B}_2$ are

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{N\times N} & \mathbf{I}_{N\times N} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}_d \end{bmatrix} \tag{3}$$

$$\mathbf{B}_1 = \begin{bmatrix} \mathbf{0}_{N\times r} \\ \mathbf{M}^{-1}\mathbf{b}_1 \end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix} \mathbf{0}_{N\times p} \\ \mathbf{M}^{-1}\mathbf{b}_2 \end{bmatrix} \tag{4}$$

3. Measurement models

Measurements $y(t)$ of the dynamic response of the structure at discrete points will be represented as linear functions of the state with additive noise $\nu(t)$ as

$$y(t) = \mathbf{C}x(t) + \mathbf{D}_1 u(t) + \mathbf{D}_2 w(t) + \nu(t) \tag{5}$$

For relative acceleration measurements $\mathbf{C} = \mathbf{c}_2[-\mathbf{M}^{-1}\mathbf{K} \quad -\mathbf{M}^{-1}\mathbf{C}_d]$, $\mathbf{D}_1 = \mathbf{c}_2\mathbf{M}^{-1}\mathbf{b}_1$, $\mathbf{D}_2 = \mathbf{c}_2\mathbf{M}^{-1}\mathbf{b}_2$; for displacement measurements $\mathbf{C} = [\mathbf{c}_2 \ \mathbf{0}_{m\times N}]$ and for velocity measurements $\mathbf{C} = [\mathbf{0}_{m\times N} \ \mathbf{c}_2]$. For displacement and velocity measurements $\mathbf{D}_1$ and $\mathbf{D}_2$ are zero matrices of appropriate dimensions. For displacement, velocity and acceleration measurements the output distribution matrix $\mathbf{c}_2 \in R^{m\times N}$ is a Boolean matrix with every row having a single non-zero entry corresponding to the degree of freedom being measured and m is the

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