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Periodic responses of a structure with 3:1 internal resonance

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ABSTRACT

This work presents a conceptually simple experiment consisting of a cantilever beam with a nonlinear spring at the tip. The configuration allows manipulation of the relative spacing between the modal frequencies of the underlying linear structure, and this permits the deliberate introduction of internal resonance. A 3:1 resonance is studied in detail; the response around the first mode shows a classic stiffening response, with the addition of more complex dynamic behaviour and an isola region. Quasiperiodic responses are also observed but in this work the focus remains on periodic responses. Predictions using Normal Form analysis and continuation methods show good agreement with experimental observations. The experiment provides valuable insight into frequency responses of nonlinear modal structures, and the implications of nonlinearity for vibration tests.

1. Introduction

There is significant research interest in the vibrations of structures that exhibit nonlinear responses. This is due to the ubiquity of such structures; for example numerous fundamental structural forms such as plates, shells and beams will exhibit nonlinear phenomena when vibrating at sufficient amplitude. Furthermore, flexible materials exhibit nonlinear stress/strain when at large strains, and mechanisms can introduce nonlinear phenomena due to geometrical effects, as well as non-smooth nonlinearities due to friction, freeplay, impact and backlash [1–3].

In addition to the academic interest in such systems, there is strong interest within industry. This is driven by the increasing demand for lightweight and flexible structures such as large wind turbine blades, or long span bridges [1]. Furthermore, new technologies such as Micro Electromechanical Systems (MEMS) utilise structures that operate on scales where effects such as electromagnetic forces generate significant nonlinear forces [4]. Nonlinearity is also being exploited in applications such as vibration isolation [5] and energy harvesting [6].

Of particular interest is the requirement to use dynamic testing methods to characterise the vibratory response of structures in order to make performance predictions, so-called system identification. While this practice has the well established methodology of modal testing in the case of linear systems [7], the presence of nonlinearity greatly complicates this task, due to the wide range of phenomena that nonlinear systems may exhibit [8,9]. Nonlinearity introduces phenomena including amplitude dependant response frequencies, super and sub harmonic responses, and multiple stable responses to a given excitation [1]. Yet more complexity emerges when multiple degrees of freedom are present, because the principle of superposition that greatly simplifies the decomposition of linear problems no longer applies. Despite this, the concept of normal modes has been extended, initially into the so-called weakly nonlinear regime [10]. The concept of

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Nomenclature

E	Young's modulus
$\vec{F}$	vector of Fourier components of the forcing signal
q_n	nth modal variable
Q_1	first modal response amplitude taken at drive frequency
Q_2	second modal response amplitude taken at third harmonic of drive frequency
$\vec{V}$	vector of Fourier components of the voltage signal sent to the shaker amplifier
u_n	resonant component of the nth modal variable
U_n	response amplitude of u_n
x	axial coordinate
z	lateral coordinate
ζ	linear modal damping ratio
θ	phase
ν	Poisson's ratio
ρ	mass density
$\phi_n(x)$	nth mode shape function
$\phi_{n,i}$, $\phi_{n,L}$	shorthand for $\phi_n(x_i)$, $\phi_n(x_L)$
ω_{nn}	linear natural angular frequency of nth mode
ω_{rn}	resonant response frequency of nth modal variable

Nonlinear Normal Modes (NNM) has since been shown to remain as an invaluable framework even when nonlinearity becomes strong [11]. Many nonlinear continuous or multi-degree of freedom systems exhibit some particularly novel responses whenever one mode of free vibration has a natural frequency that approaches an integer ratio to that of another, a condition known as internal resonance [12,11]. Forms of this behaviour have been shown in structures ranging from jointed pendulums to sagging cables [12,13].

The interest in nonlinear system identification has led to a demand for experimental demonstrators featuring continuous structures with nonlinearity; however experimental works on these types of systems are heavily outnumbered by analytical and numerical studies. In [14], Amabili presents comprehensive results on the amplitude dependence of modes of plates of different dimensions that arise due to Von Karman strains that are created by moderate amplitudes of deflection. Zaretzky and Crespo Da Silva considered one-to-one resonance between modes in different planes of a vertical cantilever subject to large amplitude and gravitational nonlinearity [15]. In a two-part paper, Rega et al. demonstrated numerous internal resonance conditions of an oscillating sagging cable [13,16]. Westra et al. use an intriguing microscopic beam with electromagnetic exciting forces to demonstrate that nonlinear effects cause one mode to affect the frequency of another [17]. Platten et al. describe a simplified experimental structure representing an aircraft wing, with two masses mounted on short beams representing pylon-mounted aircraft engines, which introduce nonlinearity via large deflections and through the joints used to attach them [18]. This structure has also been studied by Londoño et al. [19], who has also studied a nonlinear swept wing configuration [20]. Noël et al. [21] performed an experimental identification on a small space satellite structure. At the microscale, Cho et al. present an interesting study on a nonlinear coupled beam resonator, demonstrating jump phenomena and hardening softening behaviour [22].

Many further studies of this kind are based on the configuration first presented by Thouverez [23], widely known as the 'Liege beam', for example see [24,25]. This consists of a large cantilever and a far smaller cantilever, attached at the tip. The small cantilever is driven into geometrically nonlinear oscillations by the large cantilever, and may be idealised as a cubic spring attached to the tip of the large cantilever. The advantage of this approach is that the cantilever offers a straightforward underlying linear structure, which therefore greatly facilitates the recognition of nonlinear effects. Recent numerical work on this type of structure has shown that it has rich dynamics, exhibiting internal resonance effects including isolated response regions and torus bifurcations [26,27].

The current work draws on the approach of the Liege beam, but replaces the nonlinearity at the tip with a spring mechanism. This results in an experiment with straightforward underlying physics, with significant scope to adjust both the nonlinearity and the underlying linear modes of the system. The ability to adjust the underlying modes of the system means that a three-to-one internal resonance can be obtained. Stepped sine tests on this structure reveal some fascinating responses, including an experimental demonstration of an isolated region (or isola) of periodic responses, and also quasi-periodic response regions. The periodic phenomena are explained in terms of an underlying backbone structure (i.e. the free response) revealed by normal forms analysis, and show reasonable agreement with a continuation analysis of the reduced order system.

The work proceeds as follows: in Section 2 the experimental structure is described, along with discussion of how the dynamics may be tuned to internal resonance between the first two modes of the system, the linear modal properties of the beam, and the method of control. In Section 3, the decomposed equations of motion are presented, and backbone curves for the system are derived using the method of normal forms [28]. The backbone curves show that, if the first two modes are considered, three types of free resonant response are possible. In Section 4, experimental results are presented for forcing near the first modal frequency of the beam. These results fall into two groups; the first of which may be discerned with standard upward and downward stepped sine tests. The second group of results form an isola, and required a 'kick' procedure to give the initial conditions that access these results. The effect of control strategy on the accuracy of results is also described. In Section 5, the experimental results are decomposed into modal coordinates, and this is compared to results of a continuation study on the underlying modal equations. This gives the bifurcations of the forced and damped system, and

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