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Mechanical manifestations of bursting oscillations in slowly rotating systems

Zvonko Rakaric, Ivana Kovacic*

University of Novi Sad, Faculty of Technical Sciences, Centre of Excellence for Vibro-Acoustic Systems and Signal Processing, 21000 Novi Sad, Serbia

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ABSTRACT

This study is concerned with certain mechanical systems that comprise discrete masses moving along slowly rotating objects. The corresponding equation of relative motion is derived, with the rotating motion creating slowly varying external excitation. Depending on the system parameters, two cases are distinguished: two-well and single-well potential, i.e. the Duffing bistable oscillator and a pure cubic oscillator. It is illustrated that both systems can exhibit bursting oscillations, consisting of fast oscillations around the slow flow. Their mechanisms are explained in terms of bifurcation theory: the first one with respect to the existence of certain saddle-node bifurcation points, and the second one by creation of a certain hysteresis loop. The exact expressions for the slow flow are derived, in the first case as a discontinuous curve, and in the second one as a continuous curve. The influence of the excitation magnitude, which is a potential control parameter, on the characteristics of bursting oscillations is numerically illustrated.

1. Introduction

Bursting oscillations belong to the class of mixed-mode oscillations, which occur in systems with fast and slow variables. The appearance of bursting has been recognised in different fields, such as chemistry [1], biochemistry [2], physics [3] and recently mechanical energy harvesting [4]. However, bursting is central to neural phenomena, and the majority of research results have been related to excitable dynamics of neurons, so was the seminal paper written by Izhikevich [5]. Izhikevich presented the classification of bursting, noting that bursting behaviour appears in the systems governed by

$$\dot{x} = f(x, u) \qquad \text{(Fast spiking)},$$
$$\dot{u} = \mu\, g(x, u) \qquad \text{(Slow Modulation)}, \tag{1a,b}$$

where the overdot represents the derivative with respect to time t, μ is a ratio of the time scales between fast spiking and slow modulation, while $x \in R^n$ and $u \in R^k$.

Bursting oscillations have been shown to occur in non-autonomous bistable Duffing-type oscillators [6]:

$$\ddot{x} + \delta\,\dot{x} - x + x^3 = \gamma\,\cos(\omega\, t), \tag{2}$$

* Corresponding author.
E-mail addresses: zvonko@uns.ac.rs (Z. Rakaric), ivanakov@uns.ac.rs (I. Kovacic).

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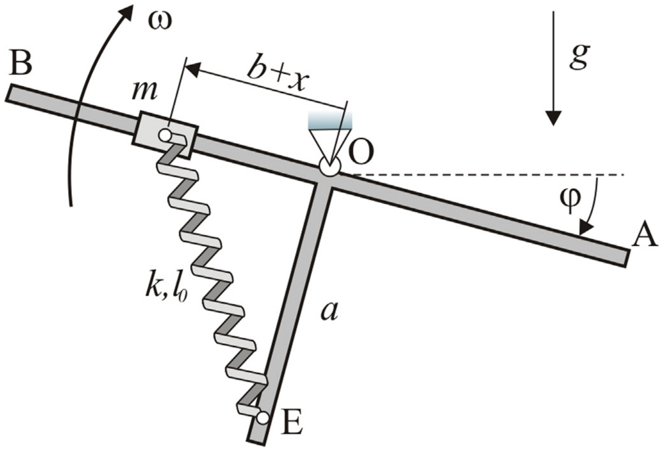

Fig. 1. Mechanical model considered.

under the condition that the external excitation has low frequency ω. This result is innovative for two reasons: this type of bistable Duffing-type oscillators is one of the classical oscillators, but have not been thoroughly investigated with respect to the behaviour caused by low-frequency excitation, and, second, it was demonstrated therein that bursting oscillations can be exhibited by non-autonomous systems, not only autonomous. Recently, the approximate analytical technique has been developed to find the solutions for bursting oscillations in terms of the slow and fast flow [7]. In addition, the model has been generalised to bistable oscillators with odd-power nonlinearity and numerical mapping was carried out to determine the range of possible dynamic behaviour for different system parameters.

Despite the clear contribution of the work presented in [6] and further enhancements and extensions given in [7], there are still several open questions, such as the links with the formal bursting classification given in [5], as well as the possibilities for mechanical manifestations of the systems governed by Eq. (2). This Short Communication aims at providing answers to these questions, with the intention to show novel, but simple mechanical manifestations that can potentially be easily realised in labs and practice. Besides these two answers, this study is to show that bursting can also occur in purely nonlinear oscillators excited at low frequency, which is a novel phenomenon in these systems.

2. Mechanical and mathematical model

The first mechanical model considered is shown in Fig. 1. It consists of a slider of mass m that can slide along a smooth straight-line part AB of a T-shaped object ABE, made of two mutually perpendicular rigid massless rods AB and OE. This object is constrained to rotate at a constant and low angular velocity ω about the horizontal axis through the hinge O. The slider is attached to the linear spring whose stiffness coefficient is k and its undeformed length is l_0. The other end of the spring is attached to the point E. The distance between O and E defines the length a.

The angle φ is the angle between the rod AB and the horizontal line, while the distance b denotes a possible stable relative equilibrium position of the slider with respect to the hinge when $\varphi=0$. Given the constant angular velocity, the angle φ changes in accordance with

$$\varphi = \omega\, t. \tag{3}$$

The distance of the slider from the hinge O is the function of the generalised coordinate x, which is, for convenience, defined as $b+x$. Before deriving the equation of motion, some remarks need to be made. The mechanism shown in Fig. 1 is associated with two motions: the rotation of the T-shaped object, and the motion of the slider along the rod AB. Thus, the square of absolute velocity of the slider stems from two mutually orthogonal components, i.e. $v^2=(b+x)^2\,\omega^2+\dot{x}^2$, so that the kinetic energy becomes

$$E_k=\frac{1}{2}\,m\,v^2=\frac{1}{2}\,m\,[\dot{x}^2+(b+x)^2\omega^2]. \tag{4}$$

The potential energy V is

$$V=mg(b+x)\sin\varphi+\frac{1}{2}k\left(\sqrt{a^2+(b+x)^2}-l_0\right)^2. \tag{5}$$

Using Eqs. (4) and 5, as well as introducing $\omega_0^2=k/m$, the following equation of motion of the slider is derived