Automatica 50 (2014) 821–831

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Adaptive output feedback tracking control of a nonholonomic mobile robot[☆]

Jiangshuai Huang [a], Changyun Wen [a], Wei Wang [b,1], Zhong-Ping Jiang [c]

[a] School of Electrical and Electronic Engineering, Nanyang Technological University, 639798, Singapore

[b] Department of Automation, Tsinghua University, Beijing, 100084, China

[c] Electrical and Computer Engineering, Polytechnical Institute of New York University, Brooklyn, NY, 11201, USA

ARTICLE INFO

Article history:
Received 6 September 2012
Received in revised form
13 October 2013
Accepted 5 December 2013
Available online 22 February 2014

Keywords:
Adaptive output feedback
Nonholonomic mobile robots
High-gain observer

ABSTRACT

An adaptive output feedback tracking controller for nonholonomic mobile robots is proposed to guarantee that the tracking errors are confined to an arbitrarily small ball. The major difficulties are caused by simultaneous existence of nonholonomic constraints, unknown system parameters and a quadratic term of unmeasurable states in the mobile robot dynamic system as well as their couplings. To overcome these difficulties, we propose a new adaptive control scheme including designing a new adaptive state feedback controller and two high-gain observers to estimate the unknown linear and angular velocities respectively. It is shown that the closed loop adaptive system is stable and the tracking errors are guaranteed to be within the pre-specified bounds which can be arbitrarily small. Simulation results also verify the effectiveness of the proposed scheme.

1. Introduction

A two-wheeled mobile robot is one of the well-known benchmark nonholonomic systems, which leads to the development of various novel controller design schemes. Much effort has been devoted to the stabilization and tracking control of this kind of nonholonomic systems, for example see Do, Jiang, and Pan (2004a,b), Fukao, Nakagawa, and Adachi (2000), Ge, Wang, and Lee (2003), and the references therein. In control theory, stabilization is usually regarded as a special case of the tracking problem. However for controlling underactuated mechanical systems with nonholonomic constraints such as nonholonomic mobile robots, the stabilization and tracking problems are totally different, and thus they are normally considered separately. For the stabilization problem, the past abundant literature aims at developing suitable discontinuous time-invariant stabilizer (Guldner & Utkin, 1994) or time-varying stabilizer (Samson, 1993) or hybrid stabilizers (Kolmanovsky & McClamroch, 1995; Sordalen & Egeland, 1995). The seminal work of Samson (1991) introduced the first time-varying feedback stabilizer for a wheeled mobile robot. For tracking control, Jiang and Nijmeijer (1997) proposed a time-varying state feedback tracking controller with backstepping technique (Krstic, Kanellakopoulos, & Kokotovic, 1995) for a kinematic model and a simplified dynamic model. Fukao et al. (2000) designed an adaptive controller based on both kinematic and dynamic models of nonholonomic mobile robots with unknown parameters. Do et al. (2004a) solved both adaptive tracking and stabilization simultaneously. Subsequent related works on the stabilization and tracking control of nonholonomic mobile robots include, but are not limited to, Dong, Huo, Tso, and Xu (2000), Egerstedt, Hu, and Stotsky (2001), Fierro and Lewis (1997), Jiang, Lefeber, and Nijmeijer (2001), Lee, Song, Lee, and Teng (2001), Luo and Tsiotras (1998) and many references therein.

It should be pointed out that most of these proposed controllers are concerned with the case that the velocities of mobile robots are measured by some devices like tachometers. But in practice such devices may not be used either because they are contaminated by noise or they are expensive. Many existing output feedback control schemes such as those proposed in Fierro (1999) and Loria (1996) for robot manipulators cannot be applied to mobile robots due to the nonholonomic constraint and certain quadratic cross terms. Jiang (2000) proposed a global output feedback control for a class of nonholonomic dynamic system, but there is not any quadratic term of unknown states contained in the system. In Do et al. (2004b), a time-varying output feedback controller was proposed for the dynamic model of a mobile robot, but it was assumed that all parameters must be known. In Loria (1996), an output feedback

☆ This work has been partially supported by NSF grants ECCS-1230040 and ECCS-1101401, and NSF of China (Grant No. 61203068). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Xiaobo Tan under the direction of Editor Miroslav Krstic.

E-mail addresses: jhuang2@e.ntu.edu.sg (J. Huang), ecywen@ntu.edu.sg (C. Wen), wwang28@tsinghua.edu.cn (W. Wang), zjiang@poly.edu (Z.-P. Jiang).

[1] Tel.: +86 10 6278 5845x283; fax: +86 10 6277 3789.

0005-1098/$ – see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.automatica.2013.12.036

controller for single-DOF Lagrangian systems was proposed with high-gain observer, but it was difficult to be extended to systems with more DOFs. To the best of our knowledge, adaptive output feedback control for nonholonomic mobile robots with unknown parameters still remains an open research problem. The main difficulty encountered lies on the simultaneous appearance of nonholonomic constraint, unknown system parameters and the coriolis matrix, which results in quadratic cross terms involving unmeasured states. With this in mind, the observer design and the parameter estimator design are intertwisted, which makes the analysis of the resulting closed loop system difficult and challenging.

In this paper, we propose a high-gain observer based scheme to address the output feedback tracking control of mobile robots in the presence of parametric uncertainties. To accomplish this, we first address the issue of designing an adaptive state feedback controller with suitable parameter estimators to achieve asymptotic tracking. Then we design two high-gain observers to estimate the unknown states and substitute the estimates to the state feedback controller and parameter estimators based on certainty equivalence principle. The stability of the system is analyzed and established by using Lyapunov approach. Due to the disturbance rejection property of the high-gain observers, it is shown that the state estimation errors are of order $O(\varepsilon)$, where ε is a small design parameter, and the tracking errors can be made arbitrary small. With our proposed control scheme, a solution is now presented for the outstanding problem of adaptive output-feedback control for nonholonomic mobile robots. Finally we demonstrate the effectiveness of our proposed controllers with simulation studies.

The rest of this paper is arranged as follows. In Section 2, we present the mobile robot model and the problem formulation. Then we proposed a new scheme to design an appropriate adaptive state feedback controller in Section 3. In Section 4, we propose two high-gain observers to estimate the linear and angular velocities respectively, and design the output feedback controller and parameter estimators. Stability is analyzed and tracking properties are also established. Simulation results are given in Section 5 and finally some concluding remarks are drawn in Section 6.

2. Robot model and problem formulation

We consider a two-wheeled mobile robot in Fig. 1 described by the following dynamic models as in Sarkar, Yun, and Kumar (1994)

$$\dot{\eta} = J(\eta)\omega, \tag{1}$$

$$M\dot{\omega} + C(\dot{\eta})\omega + D\omega = \tau, \tag{2}$$

where $\eta = (x, y, \phi)$ denotes the position and orientation of the robot, $\omega = (\omega_1, \omega_2)^T$ denotes the angular velocities of the left and right wheels, $\tau = (\tau_1, \tau_2)^T$ represents the control torques applied to the wheels, and M is a symmetric, positive definite inertia matrix, $C(\dot{\eta})$ is the centripetal and coriolis matrix, D denotes the surface friction. Matrices $J(\eta)$, M, $C(\dot{\eta})$ and D are the same as those in Do et al. (2004a), which are given below

$$J(\eta) = \frac{r}{2}\begin{bmatrix} \cos\phi & \cos\phi \\ \sin\phi & \sin\phi \\ b^{-1} & -b^{-1} \end{bmatrix}, \qquad M = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{11} \end{bmatrix},$$

$$C(\dot{\eta}) = \begin{bmatrix} 0 & c\dot{\phi} \\ -c\dot{\phi} & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix},$$

$$m_{11} = 0.25b^{-2}r^2(mb^2 + I) + I_w,$$
$$m_{12} = 0.25b^{-2}r^2(mb^2 - I),$$
$$m = m_c + 2m_w,$$
$$I = m_c d^2 + 2m_w b^2 + I_c + 2I_m,$$
$$c = 0.5b^{-1}r^2 m_c d.$$

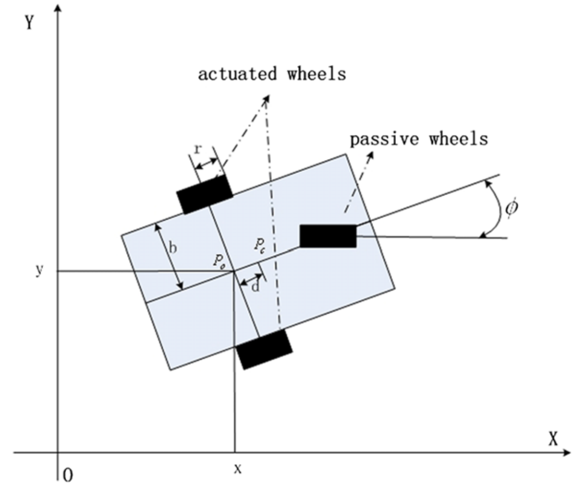

Fig. 1. A two-wheeled nonholonomic mobile robot.

In these expressions, b is the half width of the mobile robot and r is the radius of the wheel, d is the distance between the center of the mass, P_c, of the robot and the middle point between the left and right wheels, I_c, I_w and I_m are the moment of inertia of the body about the vertical axis through P_c, the wheel with a motor about the wheel axis, and the wheel with a motor about the diameter, respectively. The positive constants d_{ii}, $i = 1, 2$, are the damping coefficients. In this paper, for the sake of simplicity, we assume that the robot does not slip. Also we assume that there is no sliding between the tire and the road, i.e. there is no Coulomb-like friction.

Let v and w denote the linear and angular velocities of the robot, respectively. Then the relationship between ω_1, ω_2 and v, w is described as follows:

$$(v, w)^T = B^{-1}(\omega_1, \omega_2)^T, \qquad B = \frac{1}{r}\begin{bmatrix} 1 & b \\ 1 & -b \end{bmatrix}. \tag{3}$$

Substituting (3) into (1), we get

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \phi \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v \\ w \end{bmatrix}.$$

For the output feedback control of nonholonomic mobile robot, η is taken as the output and $\varpi = [v, w]^T$ is the state which is not directly measured in our case. Now let $(x_r, y_r, \phi_r)^T$ denote the desired reference position and orientation of the virtual robot the motion of which is described by:

$$\dot{x}_r = v_r \cos\phi_r,$$
$$\dot{y}_r = v_r \sin\phi_r,$$
$$\dot{\phi}_r = w_r, \tag{4}$$

where v_r and w_r denote the linear and angular velocities of the virtual robot. $(x_r(0), y_r(0))$ denote the initial position and $\phi_r(0)$ is the initial orientation of the reference trajectory. The control objective is to find torque τ applied to the mobile robot to ensure that the desired reference trajectories are tracked.

To achieve the objective, we need the following assumptions:

Assumption 1. $|x_r(t)| < a^\star$, $|y_r(t)| < a^\star$, where $a^\star$ is a known but arbitrary positive constant.

Assumption 2. The reference linear and angular velocities v_r and w_r and their first-order derivatives are bounded. Also there exist two positive constants $r_\star$ and T_s, such that for $T_s \le t < \infty$, $r_\star \le |v_r(t)| < \infty$.

Assumption 3. The unknown parameters of the mobile robot are in known compact sets.