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Distributed attitude synchronization control of multi-agent systems with switching topologies[☆]

Johan Thunberg [a,1], Wenjun Song [b], Eduardo Montijano [c], Yiguang Hong [b], Xiaoming Hu [a]

[a] KTH Royal Institute of Technology, S-100 44 Stockholm, Sweden

[b] Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China

[c] Centro Universitario de la Defensa and Instituto de Investigación en Ingeniería de Aragón, Zaragoza, Spain

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ABSTRACT

This paper addresses the attitude synchronization problem in multi-agent systems with directed and switching interconnection topologies. Two cases for the synchronization problem are discussed under different assumptions about the measurable information. In the first case the agents can measure their rotations relative to a global reference coordinate frame, whilst in the second case they can only measure the relative rotations between each other. Two intuitive distributed control laws based on the axis–angle representations of the rotations are proposed for the two cases, respectively. The invariance of convex balls in $SO(3)$ is guaranteed. Moreover, attitude synchronization is ensured under the well-known mild switching assumptions, the joint strong connection for the first case and joint quasi-strong connection for the second case. To show the effectiveness of the proposed control schemes, illustrative examples are provided.

1. Introduction

We address the problem of attitude synchronization. In this problem a system of rigid bodies in the three-dimensional Euclidean space shall synchronize or reach consensus in their attitudes (rotations or orientations). Or equivalently formulated, they all shall have the same rotation relative to some fixed world coordinate frame. The problem is interesting since it has applications in the real world, *e.g.*, systems of satellites, UAVs or networks of cameras. It is challenging since the kinematics are nonlinear and the system evolves on the compact manifold $SO(3)$, *i.e.*, the group of orthogonal matrices in $\mathbb{R}^{3\times 3}$ with determinant equal to 1.

As we know, connectivity is key to achieving the collective behavior in a multi-agent network. In fact, the topologies for the practical multi-agent networks may change over time. In the study of variable topologies, a well-known connectivity assumption, called (uniform) joint connection without requiring connectedness of the graph at every moment, was employed to guarantee multi-agent consensus for first-order or second-order linear or nonlinear systems (Cheng, Wang, & Hu, 2008; Hong, Gao, Cheng, & Hu, 2007; Jadbabaie, Lin, & Morse, 2003; Shi & Hong, 2009). Most existing results on the attitude synchronization problem were obtained for the case of fixed topologies.

The objective of the paper is to solve the attitude synchronization problem in continuous time systems. To provide simple and intuitive control designs, we present distributed angular velocity controllers making use of the axis–angle representation to describe the rotations of the agents. In Thunberg, Montijano, and Hu (2011) solutions for two different synchronization cases were proposed in order to synchronize the rotations. Here we largely relax the assumptions that the neighborhood or connectivity graph needs to be undirected and fixed, and consider directed and switching graphs.

We provide control laws that will lead to synchronization when the interaction graph is jointly strongly connected for the first case and jointly quasi-strongly connected in the second case. In fact, almost global attitude synchronization was achieved in Sarlette,

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E-mail addresses: johan.thunberg@math.kth.se (J. Thunberg), jsongwen@amss.ac.cn (W. Song), emonti@unizar.es (E. Montijano), yghong@iss.ac.cn (Y. Hong), hu@math.kth.se (X. Hu).

1 Tel.: +46 87906659; fax: +46 8 225320.

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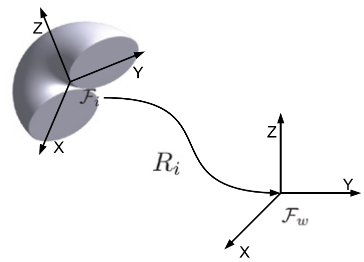

Fig. 1. A rigid body agent (agent i) here is illustrated as a half-torus. The absolute rotation or orientation of this rigid body agent is the rotation R_i of its body frame $\mathcal{F}_i$ to the fixed world frame $\mathcal{F}_W$.

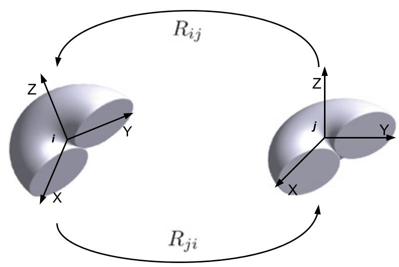

Fig. 2. Two rigid body agents, agent i and agent j, here are illustrated as half-tori. The relative rotations or orientations R_{ij} and R_{ji} between them are the rotation of $\mathcal{F}_j$ to $\mathcal{F}_i$ and the rotation from $\mathcal{F}_i$ to $\mathcal{F}_j$, respectively.

Sepulchre, and Leonard (2009) based on the joint connection, where auxiliary variables were introduced. Here the proposed controller is relatively simple in the static linear feedback form, which is very common in consensus control to make each agent move toward the average position of its neighbors.

In this work the control design is conducted on a kinematic level. A reason for this is that control laws in the robotics community often are specified on a kinematic level. The dynamic equations are platform dependent and differ between applications. We show that the well-known consensus control laws used for positional consensus of systems of agents with single integrator dynamics can be used for rotational consensus after a simple transformation, meaning that the same type of control laws can be used for rotational and positional consensus.

The paper proceeds as follows. In Section 2 we introduce the axis–angle representation and the kinematics of the agents. In Section 3 we introduce two standard cases for the general synchronization problem and propose a control scheme for each case. Sections 4 and 5 show the attitude synchronization under joint connection assumptions in the two cases, respectively. In Section 6 two illustrative examples are provided to show the synchronization when the two different controllers are used. The paper is concluded in Section 7.

2. Preliminaries

We consider a system of n agents (rigid bodies). We denote the world frame as $\mathcal{F}_W$ and the instantaneous body frame of agent i as $\mathcal{F}_i$, where $i \in \{1, 2, \dots, n\}$. For agent $i \in \{1, \dots, n\}$, let $R_i(t) \in SO(3)$ denote the rotation of $\mathcal{F}_i$ in the world frame $\mathcal{F}_W$ at time t, where $SO(3)$ represents the group of rotation matrices (Murray & Sastry, 1994). Let $R_{ij}(t) \in SO(3)$ denote the rotation of $\mathcal{F}_j$ in the frame $\mathcal{F}_i$, *i.e.*, $R_{ij}(t) = R_i^T(t)R_j(t)$, where $i, j \in \{1, 2, \dots, n\}$. We will throughout the text refer to the rotation $R_i(t)$ as the *absolute rotation* of agent i, whereas the rotation $R_{ij}(t)$ will be referred to as the *relative rotation* between agent j and agent i. The difference between the two kinds of rotations is illustrated in Figs. 1 and 2.

Let $\boldsymbol{x}_i(t)$ and $\boldsymbol{x}_{ij}(t)$ denote the axis–angle representations of the rotations $R_i(t)$ and $R_{ij}(t)$, respectively. The axis–angle representation is obtained from the logarithm as

$$\widehat{\boldsymbol{x}}_i = \log(R_i),$$
$$\widehat{\boldsymbol{x}}_{ij} = \log(R_i^T R_j),$$

where $\widehat{\boldsymbol{p}} \in so(3)$ denotes the skew symmetric matrix generated by $\boldsymbol{p} = [p_1, p_2, p_3]^T \in \mathbb{R}^3$, *i.e.*,

$$\widehat{\boldsymbol{p}} = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}. \tag{1}$$

Define the absolute state of the entire system (in terms of the axis–angle representation) as $\boldsymbol{x}(t) = [\boldsymbol{x}_1^T(t), \boldsymbol{x}_2^T(t), \dots, \boldsymbol{x}_n^T(t)]^T$. Note that

$$\boldsymbol{x}_{ij} = -\boldsymbol{x}_{ji},$$

and in general

$$\boldsymbol{x}_j - \boldsymbol{x}_i \neq \boldsymbol{x}_{ij},$$

which can be seen from the Baker–Campbell–Hausdorff formula (see Murray & Sastry, 1994). The axis–angle representation $\boldsymbol{x}_i$ of a rotation matrix R_i is unique for $\|\boldsymbol{x}_i\| < \pi$, which is almost all $SO(3)$. To be more precise, the open ball $B_\pi(I)$ in $SO(3)$ with radius π around the identity is diffeomorphic to the open ball $B_\pi(\boldsymbol{0}) = \{\|\boldsymbol{z}\| < \pi : \boldsymbol{z} \in \mathbb{R}^3\}$ via the logarithmic map and the mapping from skew symmetric matrices to $\mathbb{R}^3$. In this paper we restrict the rotations of the agents to be contained in this almost global region.

Provided that the rotations are contained in this restricted set, there is a close relationship between the Riemannian distance of two elements in $SO(3)$ and the axis–angle representation, given as follows

$$d_R(R_i, R_j) = \|\boldsymbol{x}_{ij}\|_2 = \theta_{ij},$$
$$d_R(I, R_i) = \|\boldsymbol{x}_i\|_2 = \theta_i,$$

where d_R denotes the Riemannian metric on $SO(3)$. The name axis–angle comes from the fact that the vector $\boldsymbol{x}_i$ can be equivalently written as $\boldsymbol{x}_i = \theta_i \boldsymbol{u}_i$, where $\boldsymbol{u}_i$ is the rotational axis and θ_i is the angle of rotation around the axis.

Denote the instantaneous angular velocity of $\mathcal{F}_i$ relative to $\mathcal{F}_W$ expressed in the frame $\mathcal{F}_i$ as $\boldsymbol{\omega}_i$. The kinematics of $\boldsymbol{x}_i$ is given by

$$\dot{\boldsymbol{x}}_i = L_{\boldsymbol{x}_i} \boldsymbol{\omega}_i, \tag{2}$$

where the transition matrix $L_{\boldsymbol{x}_i}$ is given by

$$L_{\boldsymbol{x}_i} = L_{\theta_i \boldsymbol{u}_i} = I_3 + \frac{\theta_i}{2}\widehat{\boldsymbol{u}}_i + \left(1 - \frac{\operatorname{sinc}(\theta_i)}{\operatorname{sinc}^2\left(\frac{\theta_i}{2}\right)}\right)\widehat{\boldsymbol{u}}_i^2. \tag{3}$$

The proof is found in Junkins and Schaub (2003). The function $\operatorname{sinc}(\beta)$ is defined so that $\beta \operatorname{sinc}(\beta) = \sin(\beta)$ and $\operatorname{sinc}(0) = 1$. It was shown in Malis, Chaumette, and Boudet (1999) that $L_{\theta_i \boldsymbol{u}_i}$ is invertible for $\theta \in (-2\pi, 2\pi)$. Note, however, that in this paper $\theta \in [0, \pi)$.

To represent the connectivity between the agents we introduce a directed graph (or digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The set $\mathcal{V} = \{1, \dots, n\}$ is the node set and the set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set. Agent i has a corresponding node $i \in \mathcal{V}$. Let $\mathcal{N}_i \in \mathcal{V}$ comprise the neighbors of agent i. The directed edge from j to i, denoted as (j, i), belongs to $\mathcal{E}$ if and only if $j \in \mathcal{N}_i$. We assume that $i \in \mathcal{N}_i$. The adjacency matrix $A = [a_{ij}]_{n \times n}$ is defined such that $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, while $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$.

A directed path of $\mathcal{G}$ is an ordered sequence of distinct nodes in $\mathcal{V}$ such that any consecutive two nodes in the sequence correspond to an edge of the digraph. An agent i is connected to an agent j if there is a directed path starting from j and ending in i. A digraph is