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Travelling and standing envelope solitons in discrete non-linear cyclic structures

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ABSTRACT

Envelope solitons are demonstrated to exist in non-linear discrete structures with cyclic symmetry. The analysis is based on the Non-Linear Schrodinger Equation for the weakly non-linear limit, and on numerical simulation of the fully non-linear equations for larger amplitudes. Envelope solitons exist for parameters in which the wave equation is focussing and they have the form of shape-conserving wave packages propagating roughly with group velocity. For the limit of maximum wave number, where the group velocity vanishes, standing wave packages result and can be linked via a bifurcation to the non-localised non-linear normal modes. Numerical applications are carried out on a simple discrete system with cyclic symmetry which can be seen as a reduced model of a bladed disk as found in turbo-machinery.

1. Introduction

Cyclic structures are of eminent relevance for a large number of disciplines in mechanical and civil engineering. Modern large scale buildings are often composed of modular, repeating elements. Rotating machinery almost always contains components with some sort of cyclic symmetry, for example bladed disks (blisks) of compressors and turbines in turbo-machinery. This study has been motivated most strongly from aeronautical engineering, where cyclic structures are of particular interest in aero-engines. The pressing need to further reduce weight leads to a number of design imperatives, like reduced clearances between rotating and stationary components or the introduction of new light-weight materials. In sum, non-linear effects, arising from material or geometric non-linearities, have to be taken into account more and more accurately in order to model the system behaviour adequately, especially when vibration is concerned. Within the field, localisation phenomena are of particular interest and growing importance.

For linear systems it is well known that deviations from homogeneity or symmetry may lead to localisation of vibration amplitude. The effect has first been observed in solid state physics and is there known as Anderson localisation [10]. Later in turbo-machinery the effect has been re-discovered and is now, with a view to the spectral perspective, most widely known as mistuning [2,13]. In the homogeneous case, there is a perfect discrete rotational symmetry, which means that each segment of the structure is mechanically identical. Then the vibration modes are doublets and can be viewed as two

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spatially phase-shifted stationary waves or two travelling waves with opposite velocities. If the discrete rotational symmetry of the system is slightly broken, one then often talks about a detuned system, the double modes split in frequency and the localisation phenomenon appears (see eg. [2,28]): spatial vibration localisation may then range from only slight inhomogeneity in the response, up to complete spatial confinement, where only a small subset of the sectors vibrates with large amplitude, while the other sectors remain virtually motionless.

This mechanism of vibration localisation, based on slight inhomogeneity, or symmetry breaking, of the otherwise linear system, very often forms the fundamental starting point for analysis into vibration response and fatigue of many components and sub-systems in turbo-machinery. This is quite a remarkable aspect, taking into account that today the components under consideration are subjected to higher and higher loads, and experience large displacements under normal operational and extreme loading conditions in failure cases.

For non-linear systems in general, localisation may occur in perfectly tuned systems, where the structural properties are spatially homogeneous, or at least invariant under a discrete symmetry transformation. There seems to be substantial debate on mechanisms underlying localisation, e.g. if there is only a single dominant localisation mechanism, in terms of mathematics or in terms of mechanics, or if there are several independent ones: while Anderson localisation or mistuning induced localisation is a linear mechanism, in other fields of physics non-linear localisation mechanisms are prevalent. For example, non-linear localisation may arise due to the dependence of the vibration mode shape on amplitude ('non-similar modes') [5,26]. Localisation may also occur through bifurcations from the main normal modes branches [5,6,21]. Those localised stationary solutions are often referred to as Intrinsic Localised Modes (ILM) or Discrete Breathers (DB) [4,25], and can be computed directly with numerical methods such as the Harmonic Balance Method (HBM) coupled with polynomial system solving methods (i.e. homotopy [21], or Groebner basis [7]). In contrast to such stationary solutions, travelling waves can also be computed using the HBM, and it has been shown that such solutions can also bifurcate into localised travelling waves solutions (see e.g. [6]). Localised motion has also been observed numerically in various granular chains taking into account Hertzian contact theory (see e.g. [24]).

Remarkably, most of the available work on non-linear vibration localisation in structural dynamics seems to be largely unrelated to the work done in many of the physics disciplines concerned with non-linear waves. While solitary waves, solitons, breathers, etc. have now been studied for decades in a large number of fields in physics, and have led to ground-breaking insight and to numerous applications, there seems to be a gap between the physics oriented disciplines and the engineering oriented disciplines. This is particularly true for structural dynamics, especially applied to mechanical engineering, or in our application case turbo-machinery, where there is only a very small number of studies about the numerical or experimental generation of solitons. Most of the studies focus on systems such as discrete lattices (e.g. [1,9,18]) or granular chains (e.g. [24]). Only a few studies are dealing with solitary waves in continuous structures, see e.g. [14–17]. The reasons underlying this scarcity of work on solitons in structural dynamics are many-fold. The main reason probably being that the deliberate generation of solitons in structures seems to be more difficult than in other media: while for many systems of physics (water waves, non-linear optics, plasma physics, etc.) the non-linear terms in the evolution equations are well known and easy to identify, for systems of solid mechanics analytical approaches are more involved, and only the advent of today's computing techniques seems to make inclusion of non-linearities a manageable and pragmatic affair.

While the importance and significance of homogeneous travelling wave modes is fully acknowledged in the field of turbo machinery, the potential role of finite-amplitude localised wave-packages does not seem to have been considered in the community. However, experimental studies such as [16] have shown that solitary waves can indeed appear in rotating structure, possibly through fluid structure interaction. The main ingredients for solitons (dispersion, non-linearities) along with some particular features (cyclic symmetry, fluid structure interaction, etc.) are also present in turbo-machinery, which make us think that solitary waves could also happen for example in a bladed disk. Taking into account such localised travelling waves could be useful for design strategy, in particular in order to avoid failure when localised vibration happens. Another application of solitary wave could concern fault detection in bladed disks, by studying the discrepancy between theoretically expected an experimentally determined solitary waves, as it is done for example for hyper-elastic elastic rods in [14].

The objective of this paper is to contribute to bridging the gap in the knowledge on soliton like states between physics and mechanical engineering. Here we will particularly focus on envelope solitons [18,9] in the context of non-linear structures with cyclic symmetry, which are of particular importance for turbo-machinery. We will show the existence of weakly and strongly non-linear envelope soliton solutions in a simple non-linear cyclic structure with discrete elements, which can be seen as a simplified model for a bladed disk structure. Using the Non-Linear Schrodinger Equation and its envelope soliton solutions as a starting point, both travelling and standing solitary vibration states are derived.

The paper is organised as follows. First we introduce our model and the techniques needed to derive envelope soliton solutions for it. We then apply the techniques and illustrate the existence of travelling envelope solitons for different parameter sets. In the following section we demonstrate the existence of standing soliton solutions and show how they can be understood to arise through bifurcations from the well-known non-linear non-localised modal solutions. Finally we conclude with a summary, critical remarks, and some potential perspectives.

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