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Brief paper

Partial state observability recovering for linear systems by additional sensor implementation[☆]

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ABSTRACT

This paper deals with the problem of additional sensor location in order to recover the observability of any given part of the state for structured linear systems. The proposed method is based on a graph-theoretic approach and assumes only the knowledge of the system's structure. We first provide new graphical necessary and sufficient conditions for the generic partial observability. Then, we study the location of additional sensors in order to satisfy the latter conditions. We provide necessary and sufficient requirements to be satisfied by these additional sensors and all their possible locations. The proposed solution is simple to implement because it is based on well-known algorithms, usually used for finding successors and predecessors of vertex subsets or on computation of maximal linkings in a digraph. All the used algorithms have polynomial complexity orders.

1. Introduction

The problem of estimating a part of the state is of great interest mainly in control applications. In this way, many works (Darouache, Pierrot, & Richard, 1999; Hautus, 1983; Karagiannis, Sassano, & Astolfi, 2009; Nikoukah, Campbell, & Delebecque, 1998; Trinh & Ha, 2000) are focused on the design of full or reduced state observers for linear systems to estimate a given part of the system's state for different control applications. A first step to the design of observer is the analysis of the partial state observability. Such analysis has been addressed in many studies that provide conditions on the state observability (Hautus, 1983; Kailath, 1980; Trentelman, Stoorvogel, & Hautus, 2001), which are now quite classic using algebraic or geometric criteria. When these conditions are not ensured, the only way to recover the observability is to add sensors. Many studies reviewed in van de Wal and de Jager (2001) deal with the selection and sensor placement, which almost all use an optimization criterion related to the observability Gramian, sensitivity functions, etc. To apply classical algebraic and geometric tools for addressing the sensors addition issue, the exact knowledge of the state space matrices characterizing the system's model is required. However, in several modelling problems, these matrices have a number of fixed zero entries determined by the physical laws while the remaining entries are not precisely known. Thus, we consider models where the fixed zeros are kept while the non-zero entries are replaced by free parameters. There are interesting works in the literature using this kind of models called structured models are related to the graph-theoretic approach and aim to analyse some system properties such as controllability and observability, or the solvability of several classical control problems including disturbance rejection (Dion, Commault, & Van der Woude, 2003). Such an approach has been also used for the sensor location studies as in Commault, Dion, and Trinh (2005) to obtain the minimal number of required additional sensors and the conditions they must satisfy in order to recover the state observability. Our work is closer to the latter reference with the originality that we try to recover the observability of a desired part of the state. This problem is more difficult than ensuring the whole state observability.

In this paper, we study the additional sensor location in order to recover the observability of a desired part of the state by using a graph-theoretic approach. The main contribution of the paper consists of three results. The first one lies in new graphical necessary and sufficient conditions for the observability of a state component based on a new kind of directed graph. The second one provides the necessary and sufficient conditions (contrary to the conditions given in Boukhobza, 2010), which must be satisfied by the locations of the additional sensors to ensure the observability of any given set of state components. More

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precisely, these conditions allow to explicit a subset of state components where it is necessary and sufficient to take additional measurements in order to recover the observability of some given state components. The third one consists of an algorithm which computes exhaustively the set of all useful sensors. An optimized iterative placement procedure for recovering the observability of a desired state component is then proposed. Compared to existing work, the paper's contribution and novelty lie in the fact that we handle the observability of a given part of the state and not of the whole state. In addition, we provide necessary and sufficient characterization of the additional sensor locations which allow to recover the observability of the given state components. In existing works, only necessary or sufficient conditions on this set of locations are given.

The paper is organized as follows: Section 2 is devoted to the problem formulation. A digraph representation of structured systems and some definitions are given in Section 3. Some preliminary results including new graphical characterization of the partial observability are presented in Section 4. The main result consisting of additional sensor implementation strategy in order to recover the observability of the desired state components, is given in Section 5. Some concluding remarks end the paper.

2. Problem statement

In this paper, we handle numerically non-specified systems:

$$\Sigma_\Lambda : \begin{cases} \dot{x}(t) = A^\lambda x(t) + B^\lambda u(t) \\ y(t) = C^\lambda x(t) + D^\lambda u(t) \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^q$ and $y \in \mathbb{R}^p$ are respectively the state vector, the input vector and the output vector. A^λ, B^λ, C^λ and D^λ represent matrices in which elements are either fixed to zero or assumed to be nonzero parameters noted λ_i. We assume that the entries of all these matrices constituting vector $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_h)^T \in \mathbb{R}^h$ can take any value in $\mathbb{R}^h$ or equivalently that parameters λ_i are free. If all parameters λ_i are numerically fixed, we obtain a so-called admissible realization of the structured system Σ_Λ. We say that a property is true generically (van der Woude, 2000) if it is true for almost all the realizations of the structured system Σ_Λ. Here, "for almost all the realizations" is to be understood (Dion et al., 2003; van der Woude, 2000) as "for all parameter values ($\Lambda \in \mathbb{R}^h$) except for those in some proper algebraic variety in the parameter space". The problem of sensor implementation in order to guarantee the generic observability of any given part of the state for the linear structured system Σ_Λ, can be written as a functional Lx of the state. It makes sense only if matrix L is composed of Euclidean vectors of $\mathbb{R}^n$ because the generic observability of Lx is equivalent to the observability of all the elements constituting a given subset of the state components.

Let us recall that the observability of state component x_i means that we can write an expression of $x_i(t)$ using only measurement vector $y(t)$, known inputs $u(t)$ and their derivatives. Let us consider $\mathbf{\Delta}$ a subset of state components $\{x_{i_1}, x_{i_2}, \ldots, x_{i_k}\}$ which are not observable, what are the necessary and sufficient locations of some additional sensors which guarantee the observability of all the elements of $\mathbf{\Delta}$? Next, we provide an optimal (in the sense that it uses a minimal number of sensors) procedure which enables to find additional sensors' configuration which guarantees the observability of all the elements of $\mathbf{\Delta}$. We denote the completed system by Σ_Λ^c with a new output vector y_{add} representing the additional sensors:

$$\Sigma_\Lambda^c : \begin{cases} \dot{x}(t) = A^\lambda x(t) + B^\lambda u(t) \\ y(t) = C^\lambda x(t) + D^\lambda u(t) \\ y_{add}(t) = H^\lambda x(t). \end{cases}$$

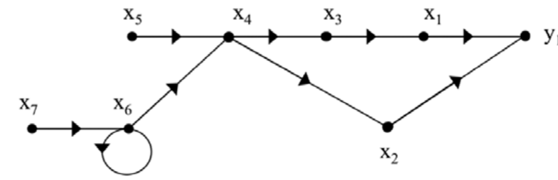

Fig. 1. Digraph associated to the system of Example 1.

For the completed system, our aim is to provide precisely and exhaustively all the possible structures of matrix H^λ which make components of $\mathbf{\Delta}$ observable for completed system Σ_Λ^c.

In order to establish the main result of the paper, we follow the steps described below.

(1) We construct a new tripartite graph associating the state components to the paths representing the observation directions in the state space, using the notion of v-disjoint paths.
(2) Using this tripartite graph, we give a characterization of the generic observability subspace dimension (Lemma 1) and then we provide new graphical necessary and sufficient conditions for the generic partial observability (Proposition 1).
(3) In the case where we have only one state component to observe, we characterize exactly all the possible locations for the additional sensor (Proposition 2). To do so, we associate to each state component $\mathbf{x_i}$ a vertex subset $\Gamma(\mathbf{x_i})$ computed iteratively from the tripartite graph. This subset contains all the useful locations to recover the observability of x_i.
(4) We extend the latter result to the case where we have to recover the observability of a state subset $\mathbf{\Delta}$ (Proposition 3). In order to propose an optimized way to add the sensors, we use the inclusion partial order relation to define $\mathbf{\Delta}^*$ a subset of $\mathbf{\Delta}$, where it is pertinent to add the measurements.
(5) Finally, we propose a practical exhaustive procedure (in the sense where all the solutions can be found) of sensors' placement, which may be iterative in its simplest and optimized form.

3. Graphical representation of structured linear systems

3.1. Digraph associated to structured linear systems

The digraph $\mathcal{G}(\Sigma_\Lambda)$ associated to Σ_Λ is constituted by a vertex set $\mathcal{V}$ and an edge set $\mathcal{E}$ *i.e.* $\mathcal{G}(\Sigma_\Lambda) = (\mathcal{V}, \mathcal{E})$. The vertices are associated to the state and the output components of Σ_Λ, $\mathcal{V} = \mathbf{X} \cup \mathbf{Y}$, where $\mathbf{X} = \{\mathbf{x_1}, \ldots, \mathbf{x_n}\}$ is the set of state vertices and $\mathbf{Y} = \{\mathbf{y_1}, \ldots, \mathbf{y_p}\}$ is the set of output vertices. The edge set is $\mathcal{E} = A\text{-edges} \cup C\text{-edges}$, with $A\text{-edges} = \{(\mathbf{x_j}, \mathbf{x_i}) \mid A^\lambda(i,j) \neq 0\}$ and $C\text{-edges} = \{(\mathbf{x_j}, \mathbf{y_i}) \mid C^\lambda(i,j) \neq 0\}$.

Hereafter, we illustrate the proposed digraph representation with an example.

Example 1. To the system defined by the following matrices, we associate the digraph in Fig. 1.

$$A^\lambda = \begin{pmatrix} 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_4 & \lambda_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_6 & \lambda_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \; C^\lambda = \begin{pmatrix} \lambda_8 & \lambda_9 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Now let us give some useful definitions and notations.

- We denote path P containing vertices $\mathbf{v_{r_0}}, \ldots, \mathbf{v_{r_k}}$ by $P = \mathbf{v_{r_0}} \to \mathbf{v_{r_1}} \to \cdots \to \mathbf{v_{r_k}}$, where $(\mathbf{v_{r_j}}, \mathbf{v_{r_{j+1}}}) \in \mathcal{E}$ for $j = 0, 1, \ldots, k-1$. We say that P covers $\mathbf{v_{r_0}}, \mathbf{v_{r_1}}, \ldots, \mathbf{v_{r_k}}$. Moreover, $\mathbf{v_{r_0}}$ is said the first vertex of P, $\mathbf{v_{r_1}}$ is the second vertex of P and so on.