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Source distance determination based on the spherical harmonics

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ABSTRACT

This paper deals with the processing of signals measured by a spherical microphone array, focusing on the utilization of near-field information of such an array. The processing, based on the spherical harmonics decomposition, is performed in order to investigate the radial-dependent spherical functions and extract their argument – distance to the source. Using the low-frequency approximation of these functions, the source distance is explicitly expressed. The source distance is also determined from the original equation (using no approximation) by comparing both sides of this equation. The applicability of both methods is first presented in the noise-less data simulation, then validated with data contaminated by the additive white noise of different signal-to-noise ratios. Finally, both methods are tested for real data measured by a rigid spherical microphone array of radius 0.15 m, consisting of 36 microphones for a point source represented by a small speaker. The possibility of determination of the source distance using low-order spherical harmonics is shown.

1. Introduction

The genuine three-dimensional symmetry of the spherical microphone array makes such an array a very strong and powerful tool capable of operating in complex reverberant fields enabling a description of a field radiated by a vibrating structure. Recently, the spherical array itself as well as the corresponding signal processing has been the subject of ongoing research. It has been advantageously utilized in sound source localization and sound field reconstruction techniques such as beamforming and near-field acoustical holography, respectively [1–4]. The analysis based on the spherical harmonics decomposition and the design of such an array is theoretically presented in a recent papers [5,6] and subsequent references. An exhaustive description of the sound field expansion in terms of the spherical harmonics can be found in [7]. Previous studies dealing with sound field analysis by a spherical array assumed the sources to be placed in the far-field of an array [8,5,9]. The capabilities of an array in its near-field have been presented in very recent papers [10,11] together with the definition of this close region, or in compact form in [12]. The capability of capturing the spherical wavefront radiated by a source (in the near-field of an array) provides additional information that can be utilized in processing.

The main objective of beamforming-based methods is the determination of the direction(s) of the arrival of sound wave(s) radiated by a far-field source(s) [13]. Such an analysis usually provides a map of sources surrounding the measurement array. On the contrary, holography-based techniques exploit the near-field information in order to increase the spatial resolution of reconstruction. Considering the interior problems, in which all the sources are located outside a certain sphere (the measurement sphere), the theory of spherical near-field acoustical holography, summarized in [7], makes the reconstruction of the surrounding sound field

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possible up to the distance of the nearest source. The *a priori* knowledge of the source position (mainly the distance from an array – in the case of a spherical array) may not be easy to achieve. For example, while attempting to reconstruct the sound field generated by a source moving in the near-field of an array [14] as close to the source as possible, it could be impractical to measure the source distance "manually". Therefore, this paper deals with the utilization of the near-field information in order to determine this distance numerically from the measured data. The idea of determination of the source distance based on the measured data appeared in [15].

The method described in this article assumes the characteristic of the sources to be similar to the point source (such as sources of radiating sections that are relatively small in relation to their operating wavelengths). The aim of this method is to determine the distances to such sources based exclusively on the measurement data. To emphasize this assumption, we will use the notation *origin of a spherical wave* instead of *the source* in the following description. It is also assumed that, in certain applications, such sources could be separated in the spatial and/or time domain by common separation methods (e.g. blind source separation, etc.). Moreover, the determined distance could be further utilized in the source extraction methods, such as the Point Source Separation (PSS) method, in which multiple incoherent point sources in a free field are assumed [16]. Once the point source locations are determined, the interfering signals could be separated by PSS. Note that in connection with a spherical array consisting usually of quite a large number of microphones (with respect to common separation/extraction method assumptions; e.g. 4 in mentioned reference), the PSS method could decompose particularly complex sound fields. An example of a real application could be the determination of the distance to the speaker's mouth, in which a manual measurement is not convenient, especially when the speaker's movement is expected.

In Section 2, processing based on spherical harmonics, which is used for the determination of the distance to the origin of a spherical wave, is briefly reviewed. In Section 3, the basic design parameters of the spherical array focusing on its near-field are discussed. Then in Section 4, the distance to the origin of a spherical wave based on the Fourier coefficients is validated using the model data set. The validation using the data measured by a rigid spherical microphone array consisting of 36 microphones is performed in Section 5 followed by discussion of the uncertainty and the Monte Carlo simulation in Section 6. Finally, the results are summarized in Section 7.

2. Spherical array signal processing

In this section, processing based on the spherical harmonics decomposition is briefly reviewed and subsequently focused on the performance in the near-field of an array.

2.1. Spherical Harmonics-based Signal Processing

Spherical harmonics decomposition represents an integral transform, in which the basis functions are represented by harmonic functions defined on the spherical surface (the spherical Fourier transform). These harmonic functions satisfy the spherical wave equation for angular variables (elevation and azimuth). Thorough description of these functions is given in [7].

Consider the acoustic pressure p to be measured on a sphere of radius $r=a$. Employing the standard spherical coordinate system (r, θ, ϕ) [7], the spectrum (marked by variable ω) of measured pressure $p(a, \theta, \phi, \omega)$ can be decomposed as

$$p\left(a, \theta, \phi, \omega\right) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} P_{mn}\left(a, \omega\right) Y_n^m\left(\theta, \phi\right), \tag{1}$$

where $Y_n^m(\theta, \phi)$ are the spherical harmonics composing an orthonormal system of functions defined using the associated Legendre functions $P_n^m(\cos\theta)$ as

$$Y_n^m\left(\theta, \phi\right) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}} P_n^m\left(\cos\theta\right) \mathrm{e}^{\mathrm{i}m\phi}, \tag{2}$$

where j stands for the imaginary unit and integer numbers n and m represent the orders and all corresponding degrees ($m \in [-n, n]$), respectively. In Eq. (1), $P_{mn}(a, \omega)$ are the Fourier coefficients given by the forward spherical Fourier transform

$$P_{mn}\left(a, \omega\right) = \int\int p\left(a, \theta, \phi, \omega\right) Y_n^{m*}\left(\theta, \phi\right) \sin\theta \mathrm{d}\theta \mathrm{d}\phi, \tag{3}$$

where the integration is performed over a sphere. The coefficients P_{mn} will be used in Section 4 for the determination of the distance to the origin of a spherical wave. The asterisk stands for the complex conjugation. Since the spherical harmonics represent the modes of a sphere, the processing is sometimes referred to as phase-mode processing in connection with beamforming techniques [8]. The radial dependence of these basis functions focusing on the region close to an array is discussed in the next section.

2.2. Near-field spherical microphone array

The transition between the near and the far field of an array is usually related to the approximation error of spherical wavefront in relation to the plane wave. Therefore, in the case of the capability of an array to capture the spherical wavefront, such information could be advantageously utilized for not only distance determination, but also for better spatial separation of multiple sources [11]. As expected, the array near field capabilities depend on its design as well as the processing frequency. The radial processing focusing