Mechanical Systems and Signal Processing 85 (2017) 1005–1019

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Experimental chaotic quantification in bistable vortex induced vibration systems

B.H. Huynh[a], T. Tjahjowidodo[b,*]

[a] *Energy Research Institute at NTU, Interdisciplinary Graduate School, Nanyang Technological University, Singapore 639798, Singapore*
[b] *School of Mechanical & Aerospace Engineering, Nanyang Technological University, Singapore 639798, Singapore*

ARTICLE INFO

Keywords:
Vortex induced vibrations (VIV)
Bistable stiffness
Chaotic vibrations
Lyapunov exponent
Surrogate data testing

ABSTRACT

The study of energy harvesting by means of vortex induced vibration systems has been initiated a few years ago and it is considered to be potential as a low water current energy source. The energy harvester is realized by exposing an elastically supported blunt structure under water flow. However, it is realized that the system will only perform at a limited operating range (water flow) that is attributed to the resonance phenomenon that occurs only at a frequency that corresponds to the fluid flow. An introduction of nonlinear elements seems to be a prominent solution to overcome the problem. Among many nonlinear elements, a bistable spring is known to be able to improve the harvested power by a vortex induced vibrations (VIV) based energy converter at the low velocity water flows. However, it is also observed that chaotic vibrations will occur at different operating ranges that will erratically diminish the harvested power and cause a difficulty in controlling the system that is due to the unpredictability in motions of the VIV structure. In order to design a bistable VIV energy converter with improved harvested power and minimum negative effect of chaotic vibrations, the bifurcation map of the system for varying governing parameters is highly on demand.

In this study, chaotic vibrations of a VIV energy converter enhanced by a bistable stiffness element are quantified in a wide range of the governing parameters, i.e. damping and bistable gap. Chaotic vibrations of the bistable VIV energy converter are simulated by utilization of a wake oscillator model and quantified based on the calculation of the Lyapunov exponent. Ultimately, a series of experiments of the system in a water tunnel, facilitated by a computer-based force-feedback testing platform, is carried out to validate the existence of chaotic responses. The main challenge in dealing with experimental data is in distinguishing chaotic response from noise-contaminated periodic responses as noise will smear out the regularity of periodic responses. For this purpose, a surrogate data test is used in order to check the hypotheses for the presence of chaotic behavior. The analyses from the experimental results support the hypothesis from simulation that chaotic response is likely occur on the real system.

1. Introduction

Vortex induced vibrations of elastically constrained structures in water flows have recently received intensive attentions as a promising and potential alternative among many available sources of renewable energy. The first vortex-induced-vibrations based energy converter trademarked as VIVACE was introduced in [1]. The VIVACE energy converter consists of a cylinder submerged

---

[*] Corresponding author.
E-mail address: ttegoeh@ntu.edu.sg (T. Tjahjowidodo).

http://dx.doi.org/10.1016/j.ymssp.2016.09.025
Received 27 April 2016; Received in revised form 8 August 2016; Accepted 17 September 2016

perpendicularly into a water flow. The cylinder is elastically supported by a linear spring and its motion is constrained along the cross-flow direction, thus allowing it to move in one degree of freedom. When the water flow passes the cylinder, it will induce periodic drag and lift forces imposing along the surface of the cylinder that is due to the occurrence of vortices alternately shedding on two sides of the wake region behind the cylinder. Under the effect of lift forces, the cylinder will vibrate in the cross-flow direction since it is constrained by the spring in this direction. These vibrations of the cylinder are known as VIV (vortex induced vibrations), whose elaborative characteristics are widely reviewed in ([2–4]). A transmission mechanism is utilized to connect the cylinder to a generator and convert the VIV into rotations of the shaft in the generator. In this way, usable energy, i.e. electrical energy, is produced from kinetic energy of water flows.

A critical parameter in designing a VIV energy converter is its natural frequency, which essentially is determined by the spring stiffness and the effective mass of the oscillating structure. In order to maximize the harvested power, the natural frequency of the system must coincide with the exciting frequency of lift forces, which is determined by velocity of the water flow. This resonance requirement will allow the VIV energy converter to be designed effectively for specific water flow conditions. This outstanding merit makes this approach for energy harvesting potential and promising. Unfortunately, another issue arises for a VIV energy converter operating in practical scenario, where water flow velocity in nature typically fluctuates severely. When the flow velocity deviates far from the resonance frequency, the effectiveness of the energy converter will drop significantly. Since this adversity was revealed, there were several theoretical and experimental studies (e.g. [5–10]), which focused on characterizing and optimizing the designing parameters of a VIV energy converter through maximizing the resonance range and increasing vibrating amplitude. The parameters include mass, damping, stiffness and surface roughness of the vibrating structure.

Among those studies, the significantly positive effect from the approach of appending a nonlinear stiffness element to the system in order to broaden the resonance range is discussed. Experimental studies in [11] and [12] have shown that a VIV energy converter with a hardening stiffness element has widened the resonance range toward the high operating working frequencies, i.e. high speed water flows. In contrast, a bistable stiffness element is capable to improve the harvested power at low operating frequencies ([13]). A numerical analysis, which will be discussed in details later in Section 2, also shows that bistable stiffness can significantly improve harvested power from a VIV energy converter at low velocity water flows. However, when parameters vary, including the water flow velocity, the harvested power might drop drastically due to the chaotic behavior that might manifest itself in the vibration response. Chaotic signal is defined as a non-periodic, unpredictable and seemingly noisy vibration [14]. It might manifest themselves in any nonlinear mechanical systems and commonly it appears in those with severe nonlinearities, such as mechanical systems with backlash ([15–17]) or those with hysteresis ([18–20]).

Chaotic vibrations are evidently undesired for the operation of a VIV energy converter because of the low harvested power and the unpredictability in motions that will result in difficulties in controlling the system. For this reason, the feasibility in application of bistable stiffness in a VIV energy converter to improve its performance requires a design for the converter that can avoid chaotic vibrations from its operating range. The prerequisite to design such a converter is the knowledge of chaotic response dependency to the governing parameters of the system.

Zhao et al. [21] studied chaotic responses of a VIV structure in laboratory scale. Initially, they measured the state variables of the system, i.e. the output displacement and lift force, on a testing apparatus. Subsequently, they confirmed the existence of chaotic vibration by prescribing the recorded displacement on a position-controlled-testing-platform and measured the lift forces. From the observation, they concluded the existence of chaotic vibrations when large discrepancies occur between the two measured lift forces. Perdikaris et al. [22] studied vibrations on a VIV system in a constant flow for varying amplitudes and fixed frequency. They observed that chaotic vibration manifests itself in a case of moderate amplitudes and confirmed the chaotic phenomenon by analyzing the frequency spectrum of the lift force, where no dominant frequency appears in the spectrum. Blackburn and Henderson [23] used the 2D numerical simulation to investigate the lock-in behavior of the cylinder excited by a constant flow. They confirmed chaotic vibrations at a certain frequency ratio by observing non-periodic vibrations and the auto-spectrum of the cross-flow vibrations, where the spectral peaks do not match to the natural frequency and the Strouhal frequency.

In this paper, chaotic vibrations of a bistable VIV energy converter will be quantified in a wide range of the two governing parameters: damping and bistable gap, which are figured out by a dimensional analysis on the system. The quantification will be mainly based on the largest Lyapunov exponent of the time series displacement of the cylinder, which can be considered as the footprint of chaos if it is positive and its magnitude measures the degree of chaos in the system ([24]). Therefore, its dependency on damping and bistable gap will serve as a guideline to design a bistable VIV energy converter.

The calculation of the largest Lyapunov exponent relies on the nonlinear time series analysis procedure, which is systematically presented in [25–27] and was successfully applied to analyze the chaotic responses of some mechanical systems in [15] and [28]. To evaluate, a time delay phase space is reconstructed from the time series data to calculate the largest Lyapunov exponent. In addition, dealing with experimental data requires a procedure to distinguish the presence of chaotic behavior in the non-linear system and of the linearly correlated noise and the *surrogate data testing* will be carried out to perform this task. This approach offers a more consistent chaotic quantifier and is capable of assessing the chaotic phenomenon from a single data set.

The paper is organized as follows: In Section 2, the wake oscillator model that is utilized to simulate the time series displacement of the structure will be presented. The expression of a bistable stiffness element applied in the VIV energy converter and the effect of bistable stiffness on performance of a VIV energy converter will also be discussed in this section. Elaborative discussion on the procedure of reconstructing the time delay phase space and calculating the largest Lyapunov exponent as well as results from this procedure can be found in Section 3. While Section 3 mainly discusses the analysis on simulated data, Section 4 presents the experimental setup of a bistable VIV system in a real water tunnel, data analysis and implementation the surrogate data testing to verify the presence of chaotic responses. Some key conclusions from this study are drawn in Section 5.