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Brief paper

Stabilization of switched continuous-time systems with all modes unstable via dwell time switching☆

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ABSTRACT

Stabilization of switched systems composed fully of unstable subsystems is one of the most challenging problems in the field of switched systems. In this brief paper, a sufficient condition ensuring the asymptotic stability of switched continuous-time systems with all modes unstable is proposed. The main idea is to exploit the stabilization property of switching behaviors to compensate the state divergence made by unstable modes. Then, by using a discretized Lyapunov function approach, a computable sufficient condition for switched linear systems is proposed in the framework of dwell time; it is shown that the time intervals between two successive switching instants are required to be confined by a pair of upper and lower bounds to guarantee the asymptotic stability. Based on derived results, an algorithm is proposed to compute the stability region of admissible dwell time. A numerical example is proposed to illustrate our approach.

1. Introduction

The stability issue is the main concern in the field of switched systems, which have been extensively studied in the literature (Branicky, 1998; Daafouz, Riedinger, & Iung, 2002; Decarlo, Branicky, Pettersson, & Lennartson, 2000; Geromel & Colaneri, 2006; Lee & Dullerud, 2007; Liberzon, 2003; Lin & Antsaklis, 2009; Margaliot, 2006; Shorten, Wirth, Mason, Wulff, & King, 2007; Sun & Ge, 2005). Most of the reported results are confined to the case where there exist stable subsystems within switched systems. In the early work, the research results mainly focused on the switched systems composed fully of stable modes (Allerhand & Shaked, 2011; Chesi, Colaneri, Geromel, Middleton, & Shorten, 2010; Hespanha, Liberzon, & Morse, 1999; Morse, 1996). In recent years, some advances have been reached to deal with the case when there exist some unstable modes such as Xiang and Xiang (2009), Xiang and Xiao (2012), Xiang, Xiao, and Iqbal (2012), Zhai, Hu, Yasuda, and Michel (2000, 2001, 2002) and Zhang and Shi (2009, 2010), but it should be noted that these results also require the existence of (at least one) stable subsystem to ensure the stability of the whole switched system. The main idea of these results is to activate the stable modes for sufficiently long to absorb the state divergence made by unstable modes. But, when all the subsystems are unstable, this promising idea obviously fails, since there exists no stable period to compensate the state divergence effect.

As is well known, even if all subsystems are unstable, one may carefully switch between unstable modes to make the switched system asymptotically stable, and how to design appropriate switching laws to stabilize the switched system composed fully of unstable subsystems is one of the most interesting and serious challenges for switched systems (Decarlo et al., 2000; Liberzon, 2003; Lin & Antsaklis, 2009; Sun & Ge, 2005). This problem has been extensively studied for years, e.g. Li, Wen, and Soh (2001), Margaliot and Langholz (2003), Pettersson (2003), Pettersson and Lennartson (2001), Wicks, Peleties, and DeCarlo (1998), most of them resort to state-dependent switching strategies such as the min-projection strategy (Pettersson & Lennartson, 2001), largest region function strategy (Pettersson, 2003), etc., but very few results focus on the time-dependent switching law particularly concerned with dwell time, which motivates the present study. Since the previous idea based on the presence of a stable subsystem is not applicable for the case with all subsystems unstable, we have to find another way to establish stability. On the other hand, since an appropriate switching law can stabilize the system, even though all subsystems are unstable, this implies that switching behaviors can also contain a good characteristic of stabilization in some circumstances, e.g. see the examples in Branicky (1998) and Sun and Ge (2005).

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For most of the previous results, the switching behavior has been viewed as a *bad* factor only destroying stability, such as the famous (average) dwell time technique (Hespanha et al., 1999; Morse, 1996). However, since an appropriate switching law can stabilize the system, it implies switching behaviors can also contain a *good* characteristic of stabilization in some circumstances. In this brief paper, when all modes are unstable, a sufficient condition ensuring the switched system asymptotically stable is proposed. Then, in order to derive computable ways to characterize the stabilization property of switching behavior and cover the results in the familiar conception called dwell time, the discretized Lyapunov function technique (Gu, Kharitonov, & Chen, 2003) is applied to the linear case. It is interesting to see that the time interval between two successive switching instants should be confined by a pair of upper and lower bounds to guarantee the asymptotic stability, which can be viewed as an extension of Allerhand and Shaked (2011), from the case composed of stable subsystems to the case fully composed of unstable subsystems. Finally, an algorithm is proposed to compute the admissible upper and lower bounds and determine the stability region for the dwell time.

This paper is organized as follows: Some preliminaries are introduced in Section 2. The stability analysis for a switched system with all subsystems unstable is presented in Section 3, and the main contributions, the computable condition for the switched linear system and computation on the stability region for the admissible dwell time are presented in Section 4. Then, a numerical example is provided in Section 5. Conclusions are given in Section 6.

2. Preliminaries

Let $\mathbb{R}$ denote the field of real numbers, $\mathbb{R}_{\geq 0}$ stand for non-negative real numbers, and $\mathbb{R}^n$ be the n-dimensional real vector space. $|\cdot|$ stands for the Euclidean norm. Class $\mathcal{K}$ is a class of strictly increasing and continuous functions $[0, \infty) \to [0, \infty)$ which is zero at zero. Class $\mathcal{K}_\infty$ denotes the subset of $\mathcal{K}$ consisting of all those functions that are unbounded. The notation $P > 0$ ($P \geq 0$) means P is real symmetric and positive definite (semi-positive definite). I stands for the identity matrix with appropriate dimension.

This paper is devoted to the study of switched nonlinear systems in the form of

$$\dot{x}(t) = f_{\sigma(t)}(x(t)) \tag{1}$$

where $x(t) \in \mathcal{X} \in \mathbb{R}^n$ is the state vector. Define index set $\mathcal{M} := \{1, 2, \ldots, N\}$, where N is the number of modes. $\sigma(t) : [0, \infty) \to \mathcal{M}$ denotes the switching function, which is assumed to be a piecewise constant function continuous from the right. $f_i : \mathbb{R}^n \to \mathbb{R}^n$ are smooth functions with $f_i(0) = 0,\ \forall i \in \mathcal{M}$, without loss of generality, the origin is not a stable (attractive) equilibrium for any modes $i \in \mathcal{M}$. The switching instants are expressed by a sequence $\mathcal{S} := \{t_0, t_1, t_2, \ldots, t_n, \ldots\}$ where t_0 denotes the initial time and t_n denotes the nth switching instant. The length between successive switching instants is denoted the dwell time $\tau_n = t_{n+1} - t_n$, $n = 0, 1, 2, \ldots$. In this work, we always assume that (1) is forward complete meaning for each $x_0 \in \mathcal{X} \in \mathbb{R}^n$ there exists a unique trajectory $x(t; x_0)$ for (1) satisfying $x(t_0) = x_0$, and we only consider non-zeno switchings (i.e., switching occurs finite times in a finite time interval). With respect to switching law $\sigma(t)$, the following stability notions are given.

Definition 1. Switched system (1) with switching law $\sigma(t)$ is said to be uniformly stable (US) with respect to $\sigma(t)$ if for $\forall \varepsilon > 0$, $\exists \delta(\varepsilon) > 0$ such that $|x(t)| < \varepsilon,\ \forall t \in [0, \infty)$ whenever $|x(0)| < \delta$. When for $\forall \delta > 0$ we have $|x(t)| < \varepsilon,\ \forall t \in [0, \infty)$ then system (1) is globally uniformly stable(GUS) with respect to $\sigma(t)$. Furthermore if system (1) is GUS and satisfies $\lim_{t\to\infty} x(t) = 0$, switched system (1) is globally uniformly asymptotically stable (GUAS) with respect to $\sigma(t)$.

The objective of this work is to propose a sufficient condition that guarantees the switched system (1) is GUAS with respect to switching law $\sigma(t)$ when all modes of (1) are unstable. Furthermore, particularly concerned with the linear case of system (1), the set of admissible switching laws that can stabilize the switched system will be ascertained in the framework of dwell time.

3. Stability analysis

It has been well recognized that the multiple Lyapunov function (MLF) $V_i(t),\ i \in \mathcal{M}$ is a popular stability analysis tool for switched systems, especially under dwell time constrained switching (Hespanha et al., 1999; Morse, 1996). At each switching instant t_n from mode i to j, the switching always causes a bounded increment of $V_i(t)$ which is described by $V_j(t_n) < \mu V_i(t_n),\ i \neq j,\ \forall i, j \in \mathcal{M}$, where $\mu > 1$. When unstable subsystems are involved, a class of Lyapunov functions $V_i(t)$ are allowed to increase with bounded increase rate as $L_{f_i} V_i(t) < \eta V_i(t)$, where $\eta > 0$ as unstable modes work. Then, both increment of $V_i(t)$ caused by activation of unstable modes and occurrence of switching will be compensated by the decrement produced by stable subsystems with a decrease rate of $L_{f_i} V_i(t) < -\lambda V_i(t)$, where $\lambda > 0$ (Xiang & Xiang, 2009; Xiang & Xiao, 2012; Xiang et al., 2012; Zhai et al., 2000, 2001, 2002; Zhang & Shi, 2009, 2010).

The above idea requires that there exists at least one stable subsystem satisfying $L_{f_i} V_i(t) < -\lambda V_i(t),\ \lambda > 0$ to compensate the increment of $V_i(t)$. But, this promising idea is obviously not applicable when all subsystems are unstable, i.e., $L_{f_i} V_i(t) < \eta V_i(t)$, $\eta > 0,\ \forall i \in \mathcal{M}$, since there exists no stable mode to be activated to compensate the increment of Lyapunov function. In this brief paper, we turn to the idea that increment of the Lyapunov function is compensated by switching behavior. Before presenting the main results, some useful functions are introduced in advance. Suppose there exists a set of continuous non-negative functions $V_i : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, and a scalar $\eta > 0$ that satisfies

$$\alpha_1(|x|) \leq V_i(t, x) \leq \alpha_2(|x|), \quad \forall i \in \mathcal{M} \tag{2}$$

$$L_{f_i} V_i(t) \leq \eta V_i(t), \quad \forall i \in \mathcal{M}. \tag{3}$$

Since each subsystem is unstable, we cannot find Lyapunov functions which are monotonically decreasing for each mode. Thus, we have to resort to Lyapunov functions allowed to increase. Formulations (2) and (3) cover various divergences of unstable modes, e.g. see Xiang and Xiang (2009), Zhai et al. (2000, 2001, 2002) and Zhang and Shi (2009, 2010). Particularly by inequality (3), this implies the value of $V_i(t)$ may increase with a bounded rate $\eta > 0$ as each unstable mode is activated. Finally, the activated mode indication functions $\theta_i(\cdot) : [0, \infty) \to \{0, 1\}$ are defined as

$$\theta_i(t) = \begin{cases} 1, & \text{if } \sigma(t) = i \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

And we define notations $V_i(t_n^-) = \lim_{t \to t_n^-} V_i(t)$, $V_i(t_n^+) = \lim_{t \to t_n^+} V_i(t)$. Now, we are ready to propose the first result in this paper.

Theorem 1. *Consider switched system (1) given a switching sequence $\mathcal{S} := \{t_0, t_1, t_2, \ldots, t_n, \ldots\}$ generated by $\sigma(t)$. If there exists a set of continuous non-negative functions $V_i : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ satisfying (2) and (3), and a constant $0 < \mu < 1$ such that*

$$V_j(t_n^+) \leq \mu V_i(t_n^-), \quad i \neq j,\ \forall i, j \in \mathcal{M} \tag{5}$$

$$\ln \mu + \eta \tau_n < 0, \quad \forall n = 0, 1, 2, \ldots \tag{6}$$

where $\tau_n = t_{n+1} - t_n$, $n = 0, 1, 2, \ldots$, then, switched system (1) is GUAS with respect to switching law $\sigma(t)$.