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Brief paper

An efficient hierarchical identification method for general dual-rate sampled-data systems[☆]

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ABSTRACT

For the lifted input–output representation of general dual-rate sampled-data systems, this paper presents a decomposition based recursive least squares (D-LS) identification algorithm using the hierarchical identification principle. Compared with the recursive least squares (RLS) algorithm, the proposed D-LS algorithm does not require computing the covariance matrices with large sizes and matrix inverses in each recursion step, and thus has a higher computational efficiency than the RLS algorithm. The performance analysis of the D-LS algorithm indicates that the parameter estimates can converge to their true values. A simulation example is given to confirm the convergence results.

1. Introduction

Multirate systems in which the inputs and outputs are scheduled at different sampling rates are common in practical industrial processes because of hardware limitations on sensory and actuating devices. For example, the D/A converters and the A/D converters are often operated at different rates to achieve better trade-offs between performance and implementation cost (Liu, Marquez, & Lin, 2008). For decades, multirate systems have received much attention. There is an extensive research literature on multirate sampling of many topics, including signal processing (Chen, 1997; Yu, Shi, & Huang, 2008), system control (Albertos & Salt, 2011; Gao, Wu, & Shi, 2009; Liu, Wang, & Wu, 2012; Zhang, Hui, & Wang, 2013; Zhang, Shi, & Saadat Mehr, 2011), fault detection (Izadi, Zhao, & Chen, 2005; Li, Shah, & Xiao, 2008; Mao, Jiang, & Shi, 2010), state estimation (Ding, Liu, Chen, & Yao, 2014; Liang, Chen, & Pan, 2009; Zhang, Basin, & Skliar, 2007) and system identification (Ding, 2013a,b; Liu & Lu, 2010). As a special class of multirate systems, dual-rate systems are often encountered in practice. There are two main sampling schemes for dual-rate systems. One is that the slow sampling period is an integer multiple of the fast period, such as the sampling patterns in Ding, Fan, and Lin (2013), Raghavan, Tangirala, Gopaluni, and Shah (2006) and Salt, Sala, and Albertos (2011). The other is that the input and output sampling periods are ph and qh, with p and q are two integers whose greatest common divisor is 1, such as the sampling patterns in Ding and Chen (2005a) and Li, Shah, and Chen (2001). To distinguish the two schemes, we term the former as the simple dual-rate sampling and the latter as the general dual-rate sampling.

The objective of dual-rate system identification is to establish a mapping relationship between the dual-rate inputs and outputs. Several identification methods and techniques have been developed for dual-rate systems. For the simple dual-rate systems in which the output sampling period is an integer multiple of the updating period, the single-rate (fast rate) system model can be directly identified from the available dual-rate data using an auxiliary model based identification algorithm (Ding & Chen, 2004). A polynomial transformation technique has been applied to obtain an equivalent dual-rate system model that does not involve the intersample output data, and a stochastic gradient algorithm

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and a least squares-based algorithm have been presented to identify the equivalent dual-rate systems in Ding, Ding, Liu, and Liu (2011), Ding, Han, and Chen (2010) and Ding, Liu, and Yang (2008). The polynomial transformation technique is convenient to transform the simple dual-rate systems whose single-rate system models are input–output representations (output error models or equation error models) (Ding & Chen, 2005b). How to use the polynomial transformation technique for modeling dual-rate systems described by state-space models is still an open research question.

The lifting technique, which can convert a multirate system into a lifted state space model with a slow period and increased dimensionality, is a standard technique to treat multirate systems (Mizumoto, Chen, Ohdaira, Kumon, & Iwai, 2007; Yu et al., 2008). The lifting technique has been widely applied to general dual-rate state-space systems (Ding & Chen, 2005a; Li et al., 2001). Based on the lifted system models, several identification methods can be employed, such as the subspace methods in Li et al. (2001) and the combined parameter and state estimation algorithm in Ding and Chen (2005a). Differing from the state space model identification in Ding and Chen (2005a) and Li et al. (2001), this paper addresses the identification problems of general dual-rate systems of lifted input–output representations based on the hierarchical identification principle (Ding & Chen, 2005c,d; Ding et al., 2014).

Using the polynomial transformation technique or the lifting technique, the obtained dual-rate system models involve more parameters than the original systems, which will unavoidably result in a heavy computational burden of the identification algorithms. This paper aims to develop a computationally efficient identification algorithm for general dual-rate systems. The hierarchical identification principle based on the decomposition is an effective method of reducing the complexity of the identification algorithms for (large-scale) multivariable systems (Ding & Chen, 2005c). The key is to decompose the identification model with a high dimensional parameter vector into several sub-models with lower dimensional parameter vectors, featuring the characteristic that the sub-models contain associated items between each other (Ding & Chen, 2005c; Ding et al., 2011). For different system identification models, the decomposition approaches are different. For example, the input–output representation of a class of multivariable systems in Ding and Chen (2005c) were decomposed into two subsystems, one with a parameter vector and the other with a parameter matrix; the dual-rate state-space model in Ding and Chen (2005a) was decomposed into several subsystems according to the lifted state-space structure; the simple dual-rate systems in Ding et al. (2011) and the non-uniformly sampled systems in Liu, Ding, and Shi (2012) were decomposed into several subsystems. In this paper, we propose a computationally efficient identification algorithm for lifted input–output relationships of general dual-rate systems partly based on the hierarchical identification principle. The main objectives are summarized as:

- to derive a lifted input–output representation for general dual-rate systems;
- to develop a computationally efficient algorithm for the obtained lifted input–output representation;
- to analyze the convergence properties of the proposed algorithm.

The rest of the paper is organized as follows. Section 2 derives the discrete-time input–output representation of general dual-rate systems. Section 3 develops a decomposition-based least squares algorithm and compares the computational load of the proposed algorithm with that of the recursive least squares algorithm. Section 4 gives the performance analysis of the proposed algorithm. Section 5 provides an illustration example. Finally, we offer some concluding remarks in Section 6.

2. System description

Consider a continuous-time system described by the state space model,

$$\begin{cases}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c\boldsymbol{x}(t) + \boldsymbol{B}_c u(t),\\ y(t) = \boldsymbol{C}\boldsymbol{x}(t) + Du(t),\end{cases} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the input and output, respectively, $\boldsymbol{A}_c$, $\boldsymbol{B}_c$ and $\boldsymbol{C}$ are constant matrices of appropriate dimensions and D is a constant. Consider the sampling pattern in Ding and Chen (2005a), where $T_1 = ph$ is the input updating period and $T_2 = qh$ is the output sampling period, with h the base period, p and q the two coprime integers, and $T := pqh = qT_1 = pT_2$ is the frame period. When $T_1 \neq T_2$, we get a general dual-rate system from (1). For every i $(0 \leqslant i \leqslant p-1)$, there exist integers $c_i \geqslant 0$ and $0 \leqslant d_i < p$ such that $iq = c_i p + d_i$. Because of using the zero-order hold, the input signal $u(t)$ takes the piecewise constant values within the updating intervals, i.e., $u(t) = u(kT + iT_1),\ kT + iT_1 \leqslant t < kT + (i+1)T_1$.

Define the lifted input and output vectors,

$$\begin{aligned}\boldsymbol{U}(kT) &:= [u(kT), u(kT+T_1), u(kT+2T_1), \ldots,\\ &\qquad u(kT+(q-1)T_1)]^{\mathrm{T}} \in \mathbb{R}^q,\\ \boldsymbol{Y}(kT) &:= [y(kT), y(kT+T_2), y(kT+2T_2), \ldots,\\ &\qquad y(kT+(p-1)T_2)]^{\mathrm{T}} \in \mathbb{R}^p.\end{aligned}$$

Referring to the method in Ding and Chen (2005a), using the lifting technique to discretize the continuous-time system in (1) with the frame period T to get the following lifted discrete-time state-space model,

$$\begin{cases}\boldsymbol{x}(kT+T) = \boldsymbol{A}_T\boldsymbol{x}(kT) + \boldsymbol{B}_T\boldsymbol{U}(kT),\\ \boldsymbol{Y}(kT) = \boldsymbol{C}_T\boldsymbol{x}(kT) + \boldsymbol{D}_T\boldsymbol{U}(kT),\end{cases} \tag{2}$$

where the structures of the involved matrices are shown in Box I.

Remark 1. The hierarchical identification method in Ding and Chen (2005a) has been presented for estimating the parameters of the general dual-rate sampled-data systems with lifted state space models in (2). The objective of this paper is to develop a new identification approach to estimate the parameters of the input–output representation corresponding to the lifted state space model in (2).

Remark 2. From the lifted state space model in (2), we can see that $\boldsymbol{D}_T$ (i.e., $\boldsymbol{\beta}(0)$) is a lower quasi-triangular matrix (i.e., certain entries in the D_T-matrix are zero), which reflects the causality constraint (Li et al., 2001). In control design and identification modeling, it is required to handle such causality constraint problems.

Remark 3. We use a continuous time SISO plant with different sampling/updating rates to derive the corresponding discrete-time representation, the parameters of the obtained model depend on the state space realization used in the model. It is worth pointing out that a state space model is not unique but its corresponding discrete-time input–output representation in (4) is always unique.

Let z^{-1} be a unit backward shift operator, z be a forward shift operator: $z^{-1}\boldsymbol{x}(kT) = \boldsymbol{x}(kT-T)$, $z\boldsymbol{x}(kT) = \boldsymbol{x}(kT+T)$, and $\boldsymbol{I}$ be an identity matrix of appropriate sizes. The dual-rate discrete-time state-space model in (2) can be transformed into an input–output representation,

$$\begin{aligned}Y(kT) &= [\boldsymbol{C}_T(z\boldsymbol{I} - \boldsymbol{A}_T)^{-1}\boldsymbol{B}_T + \boldsymbol{D}_T]\boldsymbol{U}(kT)\\ &= \frac{\boldsymbol{\beta}(z)}{\alpha(z)}\boldsymbol{U}(kT),\end{aligned}$$