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Unstable transient response of gyroscopic systems with stable eigenvalues

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ABSTRACT

Gyroscopic conservative dynamical systems may exhibit flutter instability that leads to a pair of complex conjugate eigenvalues, one of which has a positive real part and thus leads to a divergent free response of the system. When dealing with non-conservative systems, the pitch fork bifurcation shifts toward the negative real part of the root locus, presenting a pair of eigenvalues with equal imaginary parts, while the real parts may or may not be negative. Several works study the stability of these systems for relevant engineering applications such as the flutter in airplane wings or suspended bridges, brake squeal, etc. and a common approach to detect the stability is the complex eigenvalue analysis that considers systems with all negative real part eigenvalues as stable systems. This paper studies analytically and numerically the cases where the free response of these systems exhibits a transient divergent time history even if all the eigenvalues have negative real part thus usually considered as stable, and relates such a behaviour to the non orthogonality of the eigenvectors. Finally, a numerical method to evaluate the presence of such instability is proposed.

1. Introduction

In several engineering problems, the stability of gyroscopic systems is a primary concern; in fact, these systems may be characterized by the flutter instability or mode lock-in instability that is a phenomenon that can be observed when gyroscopic terms are present. Due to the variation of one or more parameters, the natural frequencies of two or more modes approach one another and coalesce, leading to an unstable behaviour.

Mode lock-in (also called modal coupling instability or flutter instability) was observed in several engineering fields such as flutter phenomena and more generally aero-elasticity in aeronautics (e.g. [1,2]), in the structural engineering field for flutter problem in cable suspended bridges [3,4], in the automotive engineering for brake squeal or windscreen chatter (see e.g. [5–13]) and in machining problems (see e.g. [14,15]). More general works tackle problems of dry contact between solid bodies [16] or gyroscopic systems with negative-definite stiffness matrices [17]. Considering specifically contact problems, mode lock-in was initially highlighted in the beam-on disk set-up, developed by Akay et al. to investigate friction driven instabilities [6]. Even if there is not an unanimous consensus in the scientific community on the causes of on brake squeal noise, several studies carried out on simplified lab set-ups have shown that squeal noise is an instability condition, that is reached when two eigenfrequencies of the system, due to the gyroscopic terms induced by friction forces, coalesce and become unstable [7–9].

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Several works study the effect of damping on the lock-in instability. In brake squeal literature, Hoffmann et al. [18] found that the presence of damping causes imperfect bifurcation with unstable eigenvalues (soothing effect), Sinou et al. [10,11,19] study the stability of a brake system taking into account the destabilizing effect of damping. Massi and Giannini [9] measured the lock-in instability and the extent of the unstable zone on the beam on disc set-up and found an experimental validation of the previous findings. Kirillov in [20] addressed the effect of damping in gyroscopic systems, while in [21] addressed a comprehensive analytical study on the interaction of eigenvalues of generic matrices relating the effect of several parameters, including damping, on the veering and the lock-in characteristics.

When dealing with gyroscopic systems, in engineering application, it is of great interest to establish design conditions that are far from the instability zone of the system: to achieve such a design, the tool usually adopted is the complex eigenvalue analysis (CEA) [7,8,10–13,19,20] that is a parametric analysis of the root locus of the eigenvalues and positive real parts are associated with the unstable behaviour of the system. The CEA is used, in practice, to detect whether a given design of the system is stable or not, and through sensitivity analysis and iterative procedures, the design is optimized to obtain a dynamic configuration of the eigenvalues that is far from possible instabilities; in this context a robust identification of the boundary of the instabilities is of primary relevance.

Goal of this paper is to show and discuss under which conditions dynamical systems, characterized by possible mode lock-in, may exhibit divergent time responses even in the presence of only negative real part eigenvalues. The study is conducted theoretically and numerically on a two degrees of freedom system, that is able to capture the dynamics of any system with two modes interacting. The results show how, considering only the real part of the eigenvalues may cause an under-estimation of the unstable zone, and introduces a novel type of instability – called transient eigenvector instability – because is due to the non orthogonality of the eigenvectors and can lead to very large free-response of the system especially in the vicinity of the lock-in and lock-out zones thus enlarging the area of the parameter space that exhibits unstable behaviour. Finally, a simple numerical way to establish the bounds of the transient eigenvector instability is proposed, and can complement the standard CEA in order to increase the robustness of the design for gyroscopic systems.

2. Lock-in in linear conservative systems

Let us consider a two degrees of freedom gyroscopic system governed by Eq. (1).

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k+\varepsilon & -\varepsilon+\alpha \\ -\varepsilon-\alpha & k_1+\varepsilon \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{1}$$

This equation is often used in literature to describe, with a low-order model, the unstable dynamics of brake squeal noise [7,10,11,19], or the aerodynamic flutter instability [2].

In this case, the eigenvalues of the system may be complex and they are given by:

$$\lambda_{1,2} = \frac{k+k_1+2\epsilon \pm \sqrt{\Delta}}{2}, \quad \Delta = (k-k_1)^2 + 4(\epsilon^2 - \alpha^2) \tag{2}$$

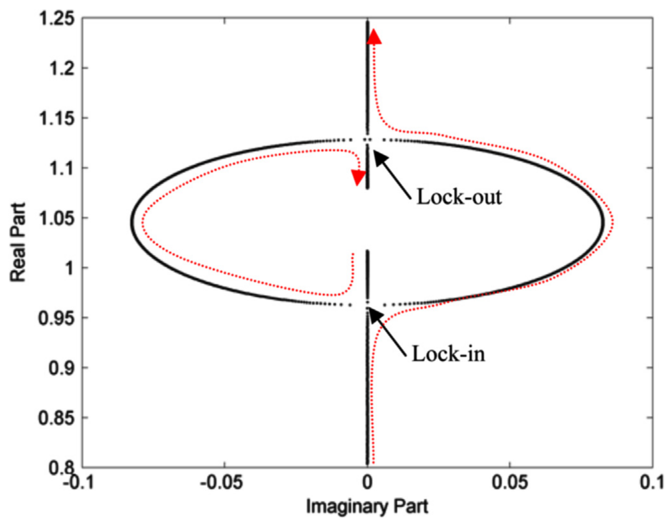

Fig. 1. Locus plot of the eigenvalues as a function on k_1: numerical value: $k=1$ N/m, $\varepsilon=0.1$ N/m, $\alpha=2\varepsilon$.