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Probabilistic updating of building models using incomplete modal data

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ABSTRACT

This paper investigates a new probabilistic strategy for Bayesian model updating using incomplete modal data. Direct mode matching between the measured and the predicted modal quantities is not required in the updating process, which is realized through model reduction. A Markov chain Monte Carlo technique with adaptive random-walk steps is proposed to draw the samples for model parameter uncertainty quantification. The iterated improved reduced system technique is employed to update the prediction error as well as to calculate the likelihood function in the sampling process. Since modal quantities are used in the model updating, modal identification is first carried out to extract the natural frequencies and mode shapes through the acceleration measurements of the structural system. The proposed algorithm is finally validated by both numerical and experimental examples: a 10-storey building with synthetic data and a 8-storey building with shaking table test data. Results illustrate that the proposed algorithm is effective and robust for parameter uncertainty quantification in probabilistic model updating of buildings.

1. Introduction

Identifying and updating the system parameters of a structural model, conditional on observed data, is a key component in structural health monitoring (SHM), since it is related to assessing the health condition, evaluating the integrity, and estimating the capacity to carry loads and risk of a structure. This topic has gained much attention recently (refer to, e.g., [1–14], among others).

In general, model updating seeks to determine a set of the most plausible parameters that best describe the structure given the measured system responses and, possibly, the external excitation. In the process of model updating, the parameters can be expressed as either specific values (deterministic) or probability distributions (probabilistic) [12]. Existing deterministic model updating strategies, such as the least squares-based methods [15,16], the heuristic algorithms [17–22], the filtering techniques [23–25], and sensitivity-based updating approaches [26–28], have been well studied and applied in SHM. Nevertheless, those strategies only find a single plausible model and have limitations in resolving issues related to model uncertainties.

Bayesian model updating techniques make possible to identify a set of plausible models with probabilistic distributions and to characterize the modeling uncertainties of a structural system. Recently, a number of Bayesian model updating

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approaches have been proposed for a reliable assessment of structural condition and a robust prediction of future structural responses. For example, Beck and Katafygiotis [29] first presented a comprehensive statistical framework for Bayesian model updating, which was then extended by their colleagues and applied to update various types of structural models using sampling techniques such as the Markov chain Monte Carlo (MCMC) simulations [30–32]. Mares et al. [33] studied stochastic model updating using a Monte-Carlo inverse procedure. Nichols et al. [34] applied the MCMC to sample the posterior parameter distributions of nonlinear structural systems and extended this approach to damage detection of composites. Beck [35] presented a rigorous framework to quantify modeling uncertainty and perform system identification using probability logic with Bayesian updating. Boulkaibet et al. [36] proposed a shadow hybrid MCMC approach for uncertainty quantification in finite element model updating. Green [37] presented a Data Annealing-based MCMC algorithm for Bayesian identification of a nonlinear dynamical system. Yan et al. [38] investigated a reverse jump MCMC method for Bayesian updating of flaw parameters.

It is quite popular to use the identified modal characteristics to update a model within the framework of Bayesian inference [39,40]. However, direct mode matching is typically required for the majority of existing Bayesian updating approaches using modal data. In practice, when incomplete measurements of mode shapes are only available, direct mode matching is not an easy task. In addition, when some of the measured modes are missing or the mode orders are unclear, direct mode matching becomes more difficult. Mode switching due to structural damage even makes the case worse [41]. Recently, Bayesian methods without requiring direct mode matching have been proposed for model updating [41–45]. This is realized through introducing the concept of system mode shapes. In the updating process, the system mode shapes become extra parameters to be updated as well. Since this method introduces additional unknown parameters into the updating process, the computational cost will increase, especially when Monte Carlo techniques are used to draw samples for the updating parameters. To alleviate this issue and to avoid direct mode matching, we propose a new strategy for Bayesian model updating using incomplete modal data. This is accomplished by employing a model reduction technique considering the available sensor locations. A MCMC simulation with adaptive random-walk steps is used to sample the posterior distributions of the model parameters.

The organization of this paper is given as follows. Section 2 presents the probabilistic model updating framework based on Bayesian inference using incomplete modal data, in which direct mode matching is not required. Modal identification as well as model reduction is also introduced. Section 3 describes the sampling technique using MCMC with adaptive random-walk steps. Sections 4 and 5 discuss numerical and experimental examples to validate the proposed model updating technique. Finally, Section 6 provides the concluding remarks of this work.

2. Probabilistic model updating without direct mode matching

The essential promise of structural model updating conditional on modal data is to modify a set of model parameters (e.g., denoted with $\theta \in \mathbb{R}^{N_\theta \times 1}$, where N_θ is the number of parameters), that minimize the discrepancy between the predicted and the measured modal quantities. The objective is to obtain an updated model which has the most probable consistency with the real structural system. Most of existing model updating strategies minimize the objective function defined as [41]

$$J(\boldsymbol{\theta}) = \sum_{i}^{N_s} \sum_{j}^{N_m} \left\{ \alpha_j \left[\omega_{i,j}^2(\boldsymbol{\theta}) - \tilde{\omega}_{i,j}^2 \right]^2 + \beta_j \left\| \boldsymbol{\phi}_{i,j}(\boldsymbol{\theta}) - \tilde{\boldsymbol{\phi}}_{i,j} \right\|_2^2 \right\} \tag{1}$$

where $\tilde{\omega}_{i,j}$ and $\tilde{\boldsymbol{\phi}}_{i,j}$ are the jth measured frequency and mode shapes of the ith data set, while $\omega_{i,j}$ and $\boldsymbol{\phi}_{i,j}$ are the corresponding predicted frequency and mode shapes from the model; α_j and β_j are the weighting coefficients; N_m is the total number of observed modes; N_s is the number of measured data sets used for model updating; and $\| \cdot \|_2$ denotes the L_2 norm of a vector. Noteworthy, there exist two major issues associated with the model updating techniques based on Eq. (1): (i) it can be seen from Eq. (1) that direct mode matching is required so as to compute $J(\boldsymbol{\theta})$; and (ii) the weighting coefficients α_j and β_j are typically empirically defined by the user, which may significantly affect the model updating result a lot. To alleviate those two issues, we propose a probabilistic strategy based on Bayesian inference for model updating using incomplete modal data, in which direct mode matching is not required.

2.1. Bayesian inference for model updating without direct mode matching

We herein consider a linear structure model with n degrees-of-freedoms (DOFs). The mass matrix $\mathbf{M} \in \mathbb{R}^{n \times n}$ is assumed to be known and the stiffness matrix $\mathbf{K} \in \mathbb{R}^{n \times n}$ is parameterized by $\boldsymbol{\theta}$, namely, $\mathbf{K} = \mathbf{K}(\boldsymbol{\theta})$. In Bayesian model updating, the posterior probability density function (PDF) of the model parameters ($\boldsymbol{\theta}$), given a specified model class, can be obtained based on the Bayes' theorem [8,30]:

$$p(\boldsymbol{\theta}|\mathcal{D}) = c^{-1} p(\mathcal{D}|\boldsymbol{\theta}) p(\boldsymbol{\theta}) \tag{2}$$

with c being the normalizing factor (the evidence given by data $\mathcal{D}$) which can be written as:

$$c = p(\mathcal{D}) = \int_{\Theta} p(\mathcal{D}|\boldsymbol{\theta}) p(\boldsymbol{\theta}) \, \mathrm{d}\boldsymbol{\theta} \tag{3}$$