ARTICLE IN PRESS

Mechanical Systems and Signal Processing ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Polynomial chaos expansion with random and fuzzy variables

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ARTICLE INFO

Article history:
Received 28 July 2015
Received in revised form
9 November 2015
Accepted 5 December 2015

Keywords:
Random systems
Structural dynamics
Polynomial chaos expansion
Fuzzy variables
Random variables
Steady-state response

ABSTRACT

A dynamical uncertain system is studied in this paper. Two kinds of uncertainties are addressed, where the uncertain parameters are described through random variables and/or fuzzy variables. A general framework is proposed to deal with both kinds of uncertainty using a polynomial chaos expansion (PCE). It is shown that fuzzy variables may be expanded in terms of polynomial chaos when Legendre polynomials are used. The components of the PCE are a solution of an equation that does not depend on the nature of uncertainty. Once this equation is solved, the post-processing of the data gives the moments of the random response when the uncertainties are random or gives the response interval when the variables are fuzzy. With the PCE approach, it is also possible to deal with mixed uncertainty, when some parameters are random and others are fuzzy. The results provide a fuzzy description of the response statistical moments.

1. Introduction

The design of vibrating structures requires an appropriate model that is often transformed to a finite element model for computation. However parameters of the model may be uncertain, and the first issue is to describe uncertainties. Some parameters may be described through a probabilistic description whereas insufficient information can be gathered for some others. The latter parameters can be modeled by fuzzy parameters [1,2], when the possible domain of variation is known; this approach has been successfully applied to determine the frequency response functions of uncertain dynamical systems [3–5]. The probabilistic approach is appealing but the statistical law is sometimes difficult to estimate. So, in practice, the laws are often supposed to follow a normal, log-normal or uniform distribution. For reliable estimate of probability distribution functions, significant amount if data is necessary. In the absence of large data, imprecise probability approaches can be used [6]. In this paper, we consider that some variables are described probabilistically while some variables are described by a fuzzy approach.

The second issue is the propagation of the uncertainties, that is to describe the responses when the parameters vary. Monte Carlo simulation (MCS) is probably the oldest method used to study uncertainty propagation. The expansion-based methods [7–10], such as the spectral approaches, are alternatives to MCS. Polynomial chaos (PC) expansion (PCE) using a

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http://dx.doi.org/10.1016/j.ymssp.2015.12.001
0888-3270/

Please cite this article as: E. Jacquelin, et al., Polynomial chaos expansion with random and fuzzy variables, Mech. Syst. Signal Process. (2015), http://dx.doi.org/10.1016/j.ymssp.2015.12.001

Galerkin scheme is one of the most widely used spectral methods. Hermite polynomials are the most used even if other orthogonal polynomials may be successfully used [11,12]. For instance, Legendre polynomials are used to propagate uniform random uncertainties. Sudret et al. made the PCE technique possible for a large number of uncertain parameters, by using an adaptive sparse PCE [13,14].

The PC method has been already applied to calculate the response of an uncertain dynamical system with fuzzy variables [15,16]. This approach requires the use of Legendre polynomials. Monti et al. used a similar approach to extend PCE to interval analysis [17].

The main objective of this paper is to give a general framework to derive the response of an uncertain linear dynamical system in terms of a PCE; the uncertainty may be described either by random variables or by fuzzy variables or by mixed uncertain variables (random and fuzzy). First background on the PCE and on fuzzy variables is given. Second, the equation satisfied by the PC component is derived. Finally examples with two uncertain parameters are studied to illustrate the method.

2. Finite element analysis with uncertain systems

The response to a dynamic excitation is modeled by a finite element approximation, where the inertia, damping and stiffness properties are assumed to be uncertain. The global matrices (mass, damping and stiffness) are obtained by the assembly procedure and the response is governed by

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{D}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{F}(t) \tag{1}$$

where **M**, **D** and **K** are the uncertain mass, damping and stiffness matrices, $\mathbf{x}(t)$ is the vector of generalized co-ordinates and **F** is the excitation vector. Matrices **M**, **D** and **K** depend on the uncertain parameters, which are either fuzzy variables or random variables. The random variables are described by the associated probability density function (pdf).

For notational convenience let **A** denote an uncertain matrix that may be **M**, **D** or **K**. In the next subsections **A** will be given for both cases of uncertainty, with **A** assumed to be given by the expansion

$$\mathbf{A}=\mathbf{A}(\mathcal{A})=\mathbf{A}_0+\sum_{i=1}^{r}a_i\mathbf{A}_i \tag{2}$$

where $\mathcal{A}=(a_1,\ldots,a_r)$ gathers all the uncertain parameters a_i.

2.1. Random variables

A is a random matrix and then uncertainties are associated with probability density functions (pdfs). Expression (2) may represent a Karhunen–Loève expansion. $\mathcal{A}=(a_1,\ldots,a_r)$ is a random vector, where a_i is a random variable with zero-mean and $\mathbf{A}_0$ is the mean matrix. However, as mentioned by Stefanou [9], the pdf of the random variables is provided by experimental measurements and often the lack of data forces assumptions to be made. Recently, Sepahvand and Marburg [18,19] proposed a new method to estimate these pdfs from vibration tests, by using the Pearson model and a generalized polynomial chaos expansion. Gaussian random variables are often chosen as a result of theoretical justification (central limit theorem, maximum entropy when the first two moments are known) and also for its simplicity. However, other distribution may be used, such as the uniform distribution, which will be used in the examples given in Section 4.

The ultimate objective of the propagation of uncertainties is to derive the pdf of the random vector $\mathbf{x}(t)$. In this paper, the first two moments of the response will be estimated for each time t in the time domain, or for each frequency f in the frequency domain.

In the following, *standard* deviates will be used. For example, if a_i has a normal distribution with variance $\sigma_{a_i}^2$, then the standard normal deviate $\xi_i=a_i/\sigma_{a_i}$ will be used. Similarly, if a_i has a uniform distribution over interval $[\min(a_i),\max(a_i)]$, then the standard uniform deviate $\xi_i=2\left(\frac{a_i-\min(a_i)}{\max(a_i)-\min(a_i)}-\frac{1}{2}\right)$ that is defined over $[-1,1]$, will be used. In both cases, the response of the uncertain system, $\mathbf{x}(t)$, depends on $\Xi=(\xi_1,\ldots,\xi_r)$.

2.2. Fuzzy variables

Fuzzy variables are used when the information on the uncertain parameters is given in terms of intervals that reflect the possible values of these parameters [5]. Fuzzy variables are related to fuzzy sets $\overline{\mathcal{F}}$, defined by a set $\mathcal{F}$ and its membership function, $\mu_{\mathcal{F}}$, defined as

$$\begin{aligned}\mu_{\mathcal{F}}&: E\to[0,1]\\ x&\mapsto\mu_{\mathcal{F}}(x)\end{aligned}$$

where E is the set of definition of the fuzzy variable and $\mathcal{F}\subset E$. The concept of fuzzy sets was introduced by Zadeh [1] who interpreted the membership function as the degree of possibility for a variable to belong to $\mathcal{F}$ (i.e. the degree of membership of x in $\mathcal{F}$). If $\mu_{\mathcal{F}}(x)=0$ then x does not belong to $\mathcal{F}$, whereas $\mu_{\mathcal{F}}(x)=1$ indicates that x belongs to $\mathcal{F}$. When $0<\mu_{\mathcal{F}}(x)<1$ then