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# Boundary control of coupled reaction–diffusion processes with constant parameters[☆]

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ABSTRACT

The problem of boundary stabilization is considered for some classes of coupled parabolic linear PDEs of the reaction–diffusion type. With reference to $n$ coupled equations, each one equipped with a scalar boundary control input, a state feedback law is designed with actuation at only one end of the domain, and exponential stability of the closed-loop system is proven. The treatment is addressed separately for the case in which all processes have the same diffusivity and for the more challenging scenario where each process has its own diffusivity and a different solution approach has to be taken. The backstepping method is used for controller design, and, particularly, the kernel matrix of the transformation is derived in explicit form of series of Bessel-like matrix functions by using the method of successive approximations to solve the corresponding PDE. Thus, the proposed control laws become available in explicit form. Additionally, the stabilization of an underactuated system of two coupled reaction–diffusion processes is tackled under the restriction that only a scalar boundary input is available. Capabilities of the proposed synthesis and its effectiveness are supported by numerical studies made for three coupled systems with distinct diffusivity parameters and for underactuated linearized dimensionless temperature-concentration dynamics of a tubular chemical reactor, controlled through a boundary at low fluid superficial velocities when convection terms become negligible.

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## 1. Introduction

The problem of boundary stabilization is considered for some classes of coupled linear parabolic Partial Differential Equations (PDEs) in a finite spatial domain $x \in [0, 1]$. Particularly, by exploiting the so-called "backstepping" approach (Krstic & Smyshlyaev, 2008; Smyshlyaev & Krstic, 2004), this work is devoted to "approximation-free" control synthesis not relying on any discretization or finite-dimensional approximation.

The backstepping-based boundary control problem for scalar heat processes was studied, e.g., in Liu (2003) and Smyshlyaev and Krstic (2004). Several classes of scalar wave processes were studied, e.g., in Krstic (2010) and Smyshlyaev and Krstic (2009), whereas complex-valued PDEs such as the Schrodinger equation were also dealt with by means of such an approach (Krstic, Guo, & Smyshlyaev, 2011). Synergies between the backstepping methodology and the flatness-based approach were studied in Meurer (2012) and Meurer and Kugi (2009) with reference to the case of spatially- and time-varying coefficients and covering spatial domains of dimension 2 and higher. In particular, in the latter situation conditions on the target system arise that somewhat resemble those considered in the remainder of the present paper. The backstepping methodology was also applied to observer design for linear parabolic PDEs with non constant coefficients in one- and multi-dimensional spatial domains Jadachowski, Meurer, and Kugi (2015) and Smyshlyaev and Krstic (2005).

More recently, high-dimensional systems of coupled PDEs are being considered in the backstepping-based boundary control setting. The most intensive efforts of the current literature are however oriented towards coupled hyperbolic processes of the transport-type (Aamo, 2013; Di Meglio, Vazquez, & Krstic, 2013; Di Meglio, Vazquez, Krstic, & Petit, 2012; Vazquez, Coron, Krstic, & Bastin, 2013; Vazquez, Krstic, & Coron, 2011). The state feedback design in Vazquez et al. (2011), which admits stabilization of $2 \times 2$ linear heterodirectional hyperbolic systems, was extended in

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Di Meglio et al. (2012) to a particular type of $3 \times 3$ linear systems, arising in modeling of multiphase flow, and to the quasilinear case in Vazquez et al. (2013). In Aamo (2013), a $2 \times 2$ linear hyperbolic system was stabilized by a single boundary control input, with the additional feature that an unmatched disturbance, generated by an *a priori* known exosystem, is rejected. In Di Meglio et al. (2013), a system of $n + 1$ coupled first-order hyperbolic linear PDEs with a single boundary input was studied.

In a recent publication (Tsubakino, Krstic, & Yamashita, 2013), two parabolic reaction–diffusion processes coupled through the corresponding boundary conditions were dealt with. The stabilization of the coupled equations is reformulated in terms of the stabilization problem for a unique process, with piecewise continuous diffusivity and (space-dependent) reaction coefficient, which can be viewed as the "cascade" between the two original systems. The problem is solved by using a unique control input acting only at a boundary. A non conventional backstepping approach with a discontinuous kernel function was employed under a certain inequality constraint involving the diffusivity parameters of the two systems and the corresponding lengths of their spatial domains.

Some specific results concerning the backstepping based boundary stabilization of parabolic coupled PDEs have additionally been presented in the literature (Aamo, Smyshlyaev, & Krstic, 2005; Vazquez & Krstic, 2010; Vazquez, Schuster, & Krstic, 2008, 2009). In Aamo et al. (2005), the Ginzburg–Landau equations, which represent a $2 \times 2$ system with equal diffusion coefficients when the imaginary and real parts are expanded, was dealt with. In Vazquez and Krstic (2010), the linearized $2 \times 2$ model of thermal-fluid convection, which entails very dissimilar diffusivity parameters, has been treated by using a singular perturbations approach combined with backstepping and Fourier series expansion. In Vazquez, Schuster et al. (2008), an observer that estimates the velocity, pressure, electric potential and current fields in a Hartmann flow was presented where the observer gains were designed using multi-dimensional backstepping. In Vazquez et al. (2009), the boundary stabilization of the linearized model of an incompressible magnetohydrodynamic flow in an infinite rectangular 3D channel, also recognized as Hartmann flow, was achieved by reducing the original system to a set of coupled diffusion equations with the same diffusivity parameter and by applying backstepping.

It is of interest to note that the multidimensional transformation considered in the present work generalizes the bi-dimensional backstepping transformation used in Aamo et al. (2005). Apart from this, the set of linear coupled kernel PDEs that was derived in Vazquez, Schuster et al. (2008); Vazquez et al. (2009) for the magnetohydrodynamic channel flow is another inspiration for the present investigation. An additional interesting feature of backstepping, which further motivates our work, is that it admits an easy synergic integration with robust control paradigms such as the sliding mode control methodology (see, e.g., Guo & Jin, 2014).

Thus motivated, the primary concern of this work is to extend the backstepping synthesis developed in Smyshlyaev and Krstic (2004), where stabilizing boundary controllers were designed for scalar unstable reaction–diffusion processes. Here, a generalization is provided by considering a set of $n$ reaction–diffusion processes, which are coupled through the corresponding reaction terms. The motivation behind the present investigation comes from chemical processes (Orlov & Dochain, 2002) where coupled temperature-concentration parabolic PDEs occur to describe the process dynamics.

A constructive synthesis procedure, with all boundary controllers given in explicit form, presents the main contribution of the paper to the existing literature. As shown in the paper, this generalization is far from being trivial because the underlying backstepping-based treatment gives rise to more complex development of finding out an explicit solution in the form of Bessel-like matrix series.

The present treatment addresses, side by side, two distinct situations which require quite different solution approaches to be adopted. First, the case where all processes have the same diffusivity ("equi-diffusivity" case, recently announced in Baccoli, Orlov, & Pisano, 2014) is attacked, and then the more challenging scenario where each process possesses its own diffusivity ("distinct-diffusivity" case) is treated. Under the requirement that the considered multi-dimensional process is fully actuated by a set of $n$ boundary control inputs acting on each subsystem, all these approaches are shown to exponentially stabilize the controlled system with an arbitrarily fast convergence rate.

Apart from this, the stabilization problem of an underactuated system of 2 coupled reaction–diffusion processes, which is relevant to regulation of tubular chemical reactors (Orlov & Dochain, 2002), is addressed under the restriction that only a unique scalar boundary input is available whereas the overall system features a certain minimum-phase property and it meets an additional restriction in the form of a suitable inequality involving both the plant and controller parameters. Exponential stability of the closed loop system is achieved in this case as well, but unlike the previously developed approaches the associated convergence rate cannot be made arbitrarily fast anymore.

The structure of the paper is as follows. In Section 2, the problem statement is presented and the underlying backstepping transformation is introduced. In Section 3, the "equi-diffusivity" scenario is investigated. Explicit solution of the kernel PDE is given for both the direct and inverse transformations, and the resulting boundary control design is presented. In Section 4, the "distinct-diffusivity" case is dealt with, which involves a simplified backstepping transformation defined by a scalar kernel function rather than a matrix one. Section 5 investigates the stabilization problem of an underactuated system of 2 coupled reaction–diffusion processes where only a unique scalar manipulable boundary input is available. Section 6 presents some simulation results. Finally, Section 7 collects concluding remarks and features future perspectives of this research.

### 1.1. Notation

The notation used throughout is fairly standard. $L_2(0, 1)$ stands for the Hilbert space of square integrable scalar functions $z(\zeta)$ on $(0, 1)$ with the corresponding norm

$$\|z(\cdot)\|_2 = \sqrt{\int_0^1 z^2(\zeta)d\zeta}. \tag{1}$$

Also, the notation

$$[L_2(0,1)]^n = \underbrace{L_2(0,1) \times L_2(0,1) \times \cdots \times L_2(0,1)}_{n\,\texttt{times}} \quad \text{and}$$

$$\|Z(\cdot)\|_{2,n} = \sqrt{\sum_{i=1}^{n} \|z_i(\cdot)\|_2^2} \tag{2}$$

is adopted for the corresponding norm of a generic vector function $Z(\zeta) = [z_1(\zeta), z_2(\zeta), \dots, z_n(\zeta)] \in [L_2(0, 1)]^n$.

$J_1(\cdot)$ and $J_2(\cdot)$ ($I_1(\cdot)$ and $I_2(\cdot)$) stand for the first and second order (modified) Bessel functions of the first kind.

With reference to a generic real-valued square matrix $A$ of dimension $n$, $S[A]$ denotes its symmetric part $S[A] = (A + A^T)/2$, and $\sigma_i(A)$ $(i = 1, 2, \dots, n)$ the corresponding eigenvalues. Provided that $A$ is also symmetric and positive definite, $\sigma_m(A)$ and $\sigma_M(A)$ denote respectively the smallest and largest eigenvalues of $A$, i.e., $\sigma_m(A) = \min_{1\le i\le n} \sigma_i(A)$, $\sigma_M(A) = \max_{1\le i\le n} \sigma_i(A)$. Finally, $I_{n\times n}$ stands for the identity matrix of dimension $n$.

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