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# Robust optimal sensor placement for operational modal analysis based on maximum expected utility

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ABSTRACT

Optimal sensor placement is essentially a decision problem under uncertainty. The maximum expected utility theory and a Bayesian linear model are used in this paper for robust sensor placement aimed at operational modal identification. To avoid nonlinear relations between modal parameters and measured responses, we choose to optimize the sensor locations relative to identifying modal responses. Since the modal responses contain all the information necessary to identify the modal parameters, the optimal sensor locations for modal response estimation provide at least a suboptimal solution for identification of modal parameters. First, a probabilistic model for sensor placement considering model uncertainty, load uncertainty and measurement error is proposed. The maximum expected utility theory is then applied with this model by considering utility functions based on three principles: quadratic loss, Shannon information, and K–L divergence. In addition, the prior covariance of modal responses under band-limited white-noise excitation is derived and the nearest Kronecker product approximation is employed to accelerate evaluation of the utility function. As demonstration and validation examples, sensor placements in a 16-degrees-of-freedom shear-type building and in Guangzhou TV Tower under ground motion and wind load are considered. Placements of individual displacement meter, velocimeter, accelerometer and placement of mixed sensors are illustrated.

## 1. Introduction

Modal parameter identification is essential for system identification, finite element model updating and damage identification [1]. In order to identify modal parameters (natural frequencies, damping ratios and mode shapes) from measured structural responses, a sensor system needs to be designed in advance. Although with the advent of sensor technology the cost of sensors is rapidly decreasing, the affordable number of sensors is still an issue in long-span bridges and super-tall buildings. When a limited number of sensors is used, inevitably uncertainties arise in the identified parameters. Many researches [2,3] have shown that optimal sensor placement (OSP) is of crucial importance in order to improve the accuracy and precision of modal parameter identification.

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OSP can be formally defined as an optimization problem consisting of performance criteria and computational issues. Among various formulations of performance criteria, information theory based approaches have attracted most attention. Kammer [2] developed the effective independence (EI) method for modal identification, which tends to maximize the determinant of the Fisher information matrix (FIM). Udwadia [3] proposed the Fisher information criterion for OSP in parameter identification, in which the optimal configuration corresponds to that maximizing the trace of the FIM. Fisher information was also introduced by Borguet and Léonard [4] in the field of engine health monitoring, where the weighted sum of the condition number, trace and determinant of the FIM was selected as the performance criterion. Papadimitriou [5] introduced the concept of information entropy for the purpose of minimizing the uncertainty in the model parameter estimation; the effect of prediction error correlation was further examined recently by Papadimitriou and Lombaert [6]. In addition, Trendafilova et al. [7] employed mutual entropy to select sensor locations to produce independent measurements. Li [8] derived the expected Kullback–Leibler divergence criterion to deploy sensors, which turns out to be identical to the weighted sum of the determinant and trace of the FIM. A comprehensive discussion and comparison of information-theory based OSP can be found in Li [9]. Aside from these information-theory based approaches, many other methods are proposed based on modal kinetic energy [10–12], model reduction [13,14] and observability [15,16]. The literature on performance criteria for OSP is so extensive that we cannot list all references. The interested reader may refer to Li [17] for additional references.

Structural responses are distinct under different input loads; thus, the optimal sensor configuration generally depends on the load case. Li [18] proposed a load-dependent sensor placement method considering both the load and structural response, and showed improved identification performance. Brehm [19] determined the optimal locations of a reference sensor under white-noise excitation and multiple impulse excitations. Furthermore, the nominal model of the structure is invariably biased, so that a nominal model-based approach cannot provide a robust design. Vinot [20] introduced a test planning procedure based on info-gap decision theory to optimize the worst possible performance for all realizations of model parameters. Castro-Triguero [21] examined the influence of model parameters on OSP using Monte Carlo simulation. The Bayesian approach has also been employed to tackle these problems. Heredia-Zavoni [22] used the expected Bayesian loss function to deploy sensors for parameter identification under seismic load considering uncertainties in stiffness. Yuen et al. [23] proposed an information entropy-based OSP methodology for modal identification considering an uncertain excitation, wherein the model parameter uncertainties were investigated by Monte Carlo sampling. Flynn [24] placed sensors in order to minimize the expected total presence of either type I or type II errors during the damage detection process using guided ultrasonic waves.

Even though great progress has been made in optimal sensor placement, many aspects of the problem remain unresolved. For example, mode-shape-based sensor placement approaches cannot incorporate prior information about natural frequencies and damping ratios. Secondly, in the formulation of FIM, independence of measured responses is assumed. This assumption obviously is not appropriate because different responses of a structure to the same excitation are naturally correlated. Thirdly, an OSP design based on a nominal model of the structure can be over-optimistic or even misleading in certain cases. Fourthly, most methods consider deployment of displacement meter, velocimeter and accelerometer to have the same effect for modal identification, but intuition suggests that this may not be true. Furthermore, although Bayesian approaches have been used [22–24], they have only employed diffuse priors, which essentially degenerates the problem into the traditional FIM-based approach. In this work, we attempt to address all the above drawbacks by utilizing the maximum expected utility theory [25], which is a perfect combination of decision theory and Bayesian statistics. Decision theory provides the mathematical foundation for selection of the optimal design, while the Bayesian approach provides a coherent framework where prior information and uncertainties regarding unknown quantities can be combined to provide the needed probabilistic information.

In this paper, we focus on OSP for operational modal analysis (OMA) [26], which due to its economy and efficiency has become the primary modal testing method in civil engineering. In OMA, only structural responses under operating conditions are measured and the unmeasured force is modeled as a broad-band random process, in most cases as a zero-mean, band-limited white-noise (BWN) process. We first develop a probabilistic framework for sensor placement incorporating model, load and measurement uncertainties, and formulate various design objectives in terms of utility functions. Computational issues are then discussed, including calculation of the covariance matrix of modal responses under BWN and fast evaluation of the utility functions. To allow the use of a linear Bayesian model, we formulate the optimization problem in terms of modal responses, which contain all the information necessary to identify modal parameters. Optimal sensor placements of an example mass-spring system and of the Guangzhou TV Tower [27] are presented in order to demonstrate the proposed approach.

## 2. Probabilistic model for sensor placement under stochastic excitation

The equation of motion of a discrete, linear, and time-invariant dynamical system with $N_d$ degrees of freedom (DOFs) under stochastic external force and ground motion is described by

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t)+\boldsymbol{C}\dot{\boldsymbol{u}}(t)+\boldsymbol{K}\boldsymbol{u}(t)=\boldsymbol{P}\boldsymbol{f}(t)-\boldsymbol{M}\boldsymbol{l}\ddot{\boldsymbol{u}}_g(t);\ \boldsymbol{u}(0)=\boldsymbol{u}_0, \dot{\boldsymbol{u}}(0)=\dot{\boldsymbol{u}}_0 \tag{1}$$

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