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A new analytical model for vibration of a cylindrical shell and cardboard liner with focus on interfacial distributed damping

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ABSTRACT

This paper proposes a new analytical model for a thin cylindrical shell that utilizes a homogeneous cardboard liner to increase modal damping. Such cardboard liners are frequently used as noise and vibration control devices for cylindrical shell-like structures in automotive drive shafts. However, most prior studies on such lined structures have only investigated the associated damping mechanisms in an empirical manner. Only finite element models and experimental methods have been previously used for characterization, whereas no analytical studies have addressed sliding friction interaction at the shell–liner interface. The proposed theory, as an extension of a prior experimental study, uses the Rayleigh–Ritz method and incorporates material structural damping along with frequency-dependent viscous and Coulomb interfacial damping formulations for the shell–liner interaction. Experimental validation of the proposed model, using a thin cylindrical shell with three different cardboard liner thicknesses, is provided to validate the new model, and to characterize the damping parameters. Finally, the model is used to investigate the effect of the liner and the damping parameters on the modal attenuation of the shell vibration, in particular for the higher-order coupled shell modes.

1. Introduction

Koruk et al. [1] recently examined the modes of a thin cylindrical shell containing a cardboard liner. This study developed controlled laboratory experiments to estimate the modal loss factors based on measured vibratory and acoustic transfer functions, and also investigated the effect of liner thickness on the natural frequencies and loss factors. However, their results are largely empirical with modal calculations provided by a finite element model. As such, this paper proposes a new analytical model for the vibratory motion of a thin cylindrical shell with a homogenous cardboard damping liner, as an extension of their prior work.

There has been considerable interest in the use of cardboard and other liners for damping of shell vibration modes, driven by the need to reduce noise and vibration from drive shafts in the automotive industry [2–5]. Koruk et al. [1] and Sun et al. [5] both cite wide usage of cardboard, rolled paper, and other hybrid treatments such as tuned absorbers to dampen beam, torsional, and shell modes. However, much of the prior literature addresses the "design-of-experiment" type of study

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and most investigations use finite element analyses with empirical damping values; a more extensive literature survey of such publications is available from Koruk et al. [1]. There is a large body of literature on constrained layer damping surface treatments [6–9], composite or sandwich structures [10], as well as foam or granular fillings [11], but no prior articles have examined relative sliding motion between material layers. Furthermore, these methods are often applied only to beams and plates [12–16] (rather than cylindrical shells), systems with only proportional damping [17], or cited in patents [18–19] with little scientific justification. In most publications, the primary damping mechanism is material (structural) damping, whereas for a cardboard liner, significant damping may arise due to friction at the sliding interface. This particular behavior has not been modeled by prior researchers, and as such, a new physics-based analytical model is needed to overcome a void in the literature.

2. Problem formulation

Consider a thin cylindrical shell of length l, thickness h, and radius a (from the central axis to the mid-plane of the shell) as depicted in Fig. 1, with the assumed cylindrical coordinate system as shown; (see Appendix A for the identification of symbols). The shell is made of an elastic material with Young's modulus E, mass density ρ, Poisson's ratio ν, and structural (material) loss factor η. The system may also have an internal cardboard liner of thickness h_L with length $l_L = l$ and radius $a_L = a - h_m$ (interference fit with sufficient "preload") where $h_m = 0.5 \cdot (h + h_L)$ is the distance between mid-surfaces of the two layers. The liner, like the shell material, is assumed to be elastic with properties E_L, ρ_L, ν_L, and η_L (though E_L and η_L may be frequency dependent). The shell–liner interface is also assumed to have associated viscous and Coulomb damping coefficients, c_ν and μ, respectively, possibly having frequency dependence as well. Finally, an external force, F_d is applied to the shell (normal to the surface) at (x_d, θ_d), and in-plane (u, v) and transverse (w) motions are observed at location (x_0, θ_0). Note that while other boundaries are possible, only free boundaries (as shown in Fig. 1) are used for this study since they are most accurately replicated experimentally.

The objectives of this paper are threefold. First, propose a new analytical model for the vibratory response of a thin cylindrical shell with a cardboard liner; this model makes several simplifying assumptions and will incorporate the liner damping into a material loss factor, η_L, along with frequency-dependent interfacial viscous and Coulomb damping effects. Second, develop a laboratory experiment to estimate parameters (beyond the prior work by Koruk et al. [1]) and to validate the proposed model in terms of eigensolutions and harmonic responses for selected modes of interest to automotive applications [2,5]. Finally, utilize this model to investigate the effects of liner properties on the damping of the system, particularly in the case of higher-order and coupled modes.

The analytical technique proposed here is the Rayleigh–Ritz method, and free boundaries are assumed at $x = 0$ and $x = L$ for $0 \le \theta < 2\pi$. A thin cardboard liner will be used for distributed damping with no adhesion, but rather an assumed interference fit with sufficient "preload" such that the liner and shell remain in contact under out-of-plane motion with $w > 0$. The cardboard used for the liners is homogenous with assumed spatially uniform and isotropic properties (though potentially spectrally-varying), rather than corrugated or other nonhomogeneous material [20]. Furthermore, the cardboard liner is assumed to have the same length as the shell to allow for more tractable analysis. Energy methods and Lagrange's equation are used to derive the governing equations in the time domain, but analyses are restricted to the frequency domain (at frequency ω [rad/s]). Frequency-dependent properties such as $E_L(\omega)$ and $\eta_L(\omega)$ may be assumed for the cardboard liner, but their temperature-dependency is beyond the scope of this work. While these liner treatments are often used for noise reduction, this study is limited to only the vibratory, rather than the acoustic, response of the system.

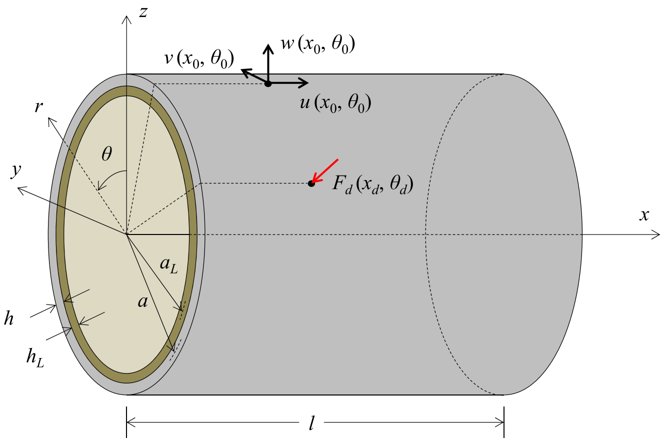

Fig. 1. Example case: cardboard liner inside a thin cylindrical shell with free boundaries; dimensions, coordinate system, harmonic force and motion locations are labeled.