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Output feedback stabilization of inverted pendulum on a cart in the presence of uncertainties☆

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ABSTRACT

An output feedback controller is proposed for stabilization of the inverted pendulum on a cart in the presence of uncertainties. The output feedback controller has a multi-time-scale structure in which Extended High-Gain Observers are used to estimate system states and uncertainties in the first and fastest time scale; dynamic inversion is used to deal with uncertain input coefficients in the second time scale; the pendulum converges to a reference trajectory in the third time scale; and finally, the reference trajectory is designed such that both the pendulum and the cart converge to the desired equilibrium in the fourth and slowest time scale. The multi-time-scale structure allows independent analysis of the dynamics in each time scale and singular perturbation methods are effectively utilized to establish exponential stability of the equilibrium. Simulation results indicate that the output feedback controller provides a large region of attraction and experimental results establish the feasibility of practical implementation.

1. Introduction

An inverted pendulum on a cart is a classical example of an underactuated mechanical system and its stabilization problem has been investigated by many researchers. Based on linearized system dynamics, controllers can be designed to stabilize the equilibrium but the size of the region of attraction is typically small. Furthermore, these controllers are not very effective in the presence of significant uncertainties in the system model. In this paper we present an output feedback control design that can stabilize the equilibrium in the presence of significant uncertainties and provide a large region of attraction.

One representative approach for stabilization of the inverted pendulum on a cart is based on the energy of the system. Spong and Praly (1996) used partial feedback linearization to linearize the cart dynamics followed by transfer of energy from the cart to the pendulum. A stabilizing controller is invoked when the configuration of the system reaches a neighborhood of the equilibrium. Astrom and Furuta (2000) used a Lyapunov function based on the potential energy of the pendulum, and Lozano, Fantoni, and Block (2000) stabilized the pendulum around its homoclinic orbit prior to stabilization. Fradkov (1996) developed a control method using an energy-based objective function and the speed-gradient, and Shiriaev, Egeland, Ludvigsen, and Fradkov (2001) proposed a modified controller using the idea of variable structure systems. Muralidharan, Ravichandran, and Mahindrakar (2009) designed a controller for the two-wheeled inverted pendulum using the interconnection and damping-passivity-based control (IDA-PBC) method proposed by Ortega, Spong, Gomez-Estern, and Blankenstein (2002) for underactuated systems. Sarras, Acosta, Ortega, and Mahindrakar (2013) combined the approach of the Immersion and Invariance proposed by Astolfi, Karagiannis, and Ortega (2007) with the Hamiltonian formulation to accommodate underactuation degree greater than one. Bloch, Chang, Leonard, and Marsden (2001); Bloch, Leonard, and Marsden (2000) used the controlled Lagrangian approach to derive a desired closed-loop system dynamics for stabilization. The controller is designed by matching the dynamic equations for the uncontrolled and controlled Lagrangians. In Bloch et al. (2000), only the kinetic energy was shaped to obtain the desired dynamics whereas both kinetic and potential energies were shaped in Bloch et al. (2001). Angeli (2001) developed a smooth feedback law for almost-global stabilization based on the energy-shaping control strategy in

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Bloch et al. (2000). Auckly, Kapitanski, and White (2000) derived a stabilizing controller by solving a set of linear partial differential equations; these equations were obtained by matching the desired closed-loop system dynamics based on the potential energy with the original dynamics.

Among other approaches, Mazenc and Praly (1996) and Teel (1996) developed control methods based on the concept of interconnected systems. In Mazenc and Praly (1996), the stability analysis was carried out using a Lyapunov function whereas in Teel (1996) a nonlinear small gain theorem was used. Olfati-Saber (2002) proposed a transformation to convert the system into cascade normal form, for which existing control methods can be used for stabilization. A two-time-scale approach was proposed by Getz and Hedrick (1995) and Srinivasan, Huguenin, and Bonvin (2009). In Getz et al. (1995), the trajectories of the pendulum were rapidly converged to a reference trajectory and the reference trajectory was slowly varied to converge the cart to its desired position. In Srinivasan et al. (2009), low gains were used near the equilibrium for separation of time scales. All of the methods discussed above require exact knowledge of the system dynamics and are unlikely to guarantee stabilization in the presence of significant uncertainties.

To deal with uncertainties of the system model, Ravichandran and Mahindrakar (2011) used a two-time-scale approach together with Lyapunov redesign. However, the transient behavior of the fast system was not analyzed. Park and Chwa (2009) utilized two sliding surfaces for the pendulum and cart subsystems to stabilize the system in the presence of disturbances but uncertainties in system parameters were not considered. Adhikary and Mahanta (2013) used backstepping and sliding mode control to the normal form of the system. Both uncertainties and disturbances were considered but they were introduced after the system was converted into normal form. Xu, Guo, and Lee (2013) used integral sliding-mode control (Cao & Xu, 2004) to deal with uncertainties in the two-wheeled mobile inverted pendulum but the size of the region of attraction of the equilibrium is small since the controller is designed based on the linearized system dynamics.

In this paper we present an output feedback controller to stabilize the inverted pendulum on a cart in the presence of significant uncertainties. Extended High-Gain Observers and dynamic inversion are combined together with a multi-time-scale structure to deal with model uncertainties. The stability analysis for the multi-time-scale structure is carried out using singular perturbation methods; the advantage of this approach is that the behavior of the system can be analyzed independently for each time scale. The multi-time-scale structure of the controller effectively provides a large region of attraction and this is illustrated through simulations. Output feedback control of the inverted pendulum on a cart has not been proposed earlier and it is shown here that it can recover the performance of the system under state feedback.

The paper is organized as follows. In Section 2, a state feedback controller is designed using a two-time-scale structure; uncertainties are not considered. In Section 3, the output feedback controller is designed in the presence of uncertainties. Simulation and experimental results are presented in Section 4 and conclusions are provided in Section 5.

2. Stabilization in the absence of uncertainties

We present a control strategy to stabilize the desired equilibrium of the inverted pendulum on a cart system, in the absence of uncertainties. The controller is based on the designs proposed by Getz (1995) and Srinivasan et al. (2009); here we cast the closed-loop system dynamics in two-time scale format for the purpose of stability analysis. The stability analysis is done by transforming the system into a standard singularly perturbed one.

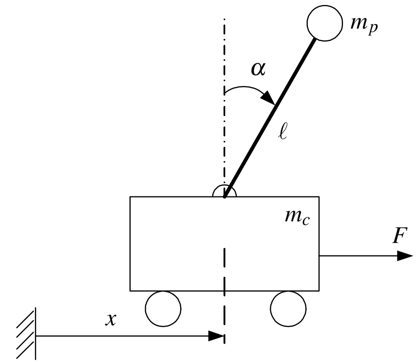

Fig. 1. Inverted pendulum on a cart.

Remark 1. As an intermediate step for the output feedback controller in Section 3, we design a controller in this section in the absence of uncertainties.

2.1. Dynamics of an inverted pendulum on a cart

The dynamics of an inverted pendulum on a cart are given by

$$\begin{bmatrix} m_p + m_c & \ell m_p \cos\alpha \\ \ell m_p \cos\alpha & \ell^2 m_p \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} \ell m_p \dot{\alpha}^2 \sin\alpha \\ g \ell m_p \sin\alpha \end{bmatrix} + \begin{bmatrix} F \\ 0 \end{bmatrix} \quad (1)$$

where m_p, m_c are masses of the pendulum and the cart, respectively; g is the acceleration due to gravity; and ℓ is the length of the pendulum—see Fig. 1. The variables x and α denote the position of the cart and the angular displacement of the pendulum, respectively; α is measured clockwise from the vertical following the notation in Getz (1995). The variable F denotes the force applied on the cart and is the control input. With the choice of state variables

$$x_1 = x, \quad x_2 = \dot{x}, \quad \alpha_1 = \alpha, \quad \alpha_2 = \dot{\alpha}$$

the system equations of (1) take the form

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = f_x(\alpha_1, \alpha_2, F), \quad \dot{\alpha}_1 = \alpha_2, \quad \dot{\alpha}_2 = f_\alpha(\alpha_1, \alpha_2, F) \quad (2)$$

where

$$\begin{aligned} f_x &= \frac{1}{(m_p + m_c - m_p\cos^2\alpha_1)} F + G_x \\ f_\alpha &= \frac{-\cos\alpha_1}{\ell(m_p + m_c - m_p\cos^2\alpha_1)} F + G_\alpha \\ G_x &= \frac{(\ell m_p \alpha_2^2 \sin\alpha_1 - m_p g \cos\alpha_1 \sin\alpha_1)}{(m_p + m_c - m_p\cos^2\alpha_1)} \\ G_\alpha &= \left(\frac{g}{\ell}\right)\sin\alpha_1 - \frac{\cos\alpha_1}{\ell} G_x. \end{aligned} \quad (3)$$

We consider equations in (2) over the domain $x = [x_1, x_2]^T \in D_x$ and $\alpha = [\alpha_1, \alpha_2]^T \in D_\alpha$ where $D_x = \{-a_{x_1} < x_1 < a_{x_1}\} \times \{-a_{x_2} < x_2 < a_{x_2}\} \subset \mathbf{R}^2$ and $D_\alpha = \{-a_{\alpha_1} < \alpha_1 < a_{\alpha_1}\} \times \{-a_{\alpha_2} < \alpha_2 < a_{\alpha_2}\} \subset \mathbf{R}^2$ are bounded. The constants, a_{x_1}, a_{x_2}, a_{α_1}, and a_{α_2} are positive numbers and $a_{\alpha_1} < \pi/2$.

2.2. Control design

The choice of the control input

$$F = (m_c + m_p - m_p\cos^2\alpha_1)(u - G_x) \quad (4)$$

with

$$u = g\tan\alpha_1 - \left(\frac{\ell}{\cos\alpha_1}\right)\nu_d \quad (5)$$