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Direct method for second-order sensitivity analysis of modal assurance criterion

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ABSTRACT

A Lagrange direct method is proposed to calculate the second-order sensitivity of modal assurance criterion (MAC) values of undamped systems. The eigenvalue problem and normalizations of eigenvectors, which augmented by using some Lagrange multipliers, are used as the constraints of the Lagrange functional. Once the Lagrange multipliers are determined, the sensitivities of MAC values can be evaluated directly. The Lagrange direct method is accurate, efficient and easy to implement. A simply supported beam is utilized to check the accuracy of the proposed method. A frame is adopted to validate the predicting capacity of the first- and second-order sensitivities of MAC values. It is shown that the computational costs of the proposed method can be remarkably reduced in comparison with those of the indirect method without loss of accuracy.

1. Introduction

Frequencies and mode shapes, which are normally calculated by numerical methods or extracted from measured experimental data, stand for the dynamic characteristics and therefore are of fundamental importance for structural dynamics. In order to get the modal pairing based on the eigenvalues extracted from the analytical and measured results, many criteria have been developed, among which the modal assurance criterion (MAC) [1,2] has been demonstrated to be extremely useful and has been developed into a mature technology applied successfully for enterprise-level finite element models. Furthermore, these MAC values have become an integral part of many design methodologies, including dynamic modification [3,4], model updating [5–8], structural health monitoring [9] and structural optimization [10,11]. The application often requires the sensitivity of the MAC values with respect to the design parameters. In addition, it was pointed out by Mottershead et al. [5] that the sensitivity based method is probably the most successful of the many techniques to the problem of model updating. With the recent developments in the techniques of model updating and dynamical optimizations, some existing commercial software packages using various algorithms have been developed to calculate the sensitivity of the MAC values by considering the indirect method, such as LMS Virtual. Lab and FEMtools. The indirect method to calculate the sensitivity of the MAC values is based on the existing sensitivity of mode shapes. Several methods have been developed for the calculation of the sensitivity of mode shapes, including the modal method [12–17], Nelson's method

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[18–24], the algebraic method [25–28], and the iterative method [29–31]. Li et al. [32] surveyed the various methods for the evaluation of the sensitivity of MAC values by classifying these methods as the indirect and direct methods. Since many design variables are considered in enterprise-level finite element models and the diagonal elements of the MAC sensitivity matrix are usually considered in the optimization problem, a direct method is presented to efficiently and accurately calculate the sensitivity of the MAC values since the method calculates the sensitivity of MAC values directly, regardless of the number of design variables. Recently, Adhikari [33,34] gave a systematical review on the sensitivity of eigenpairs for both undamped and damped systems. More recently, Lei et al. [35] studied the sensitivity of the MAC values of the damped system by using the direct method. So far, the direct method is restricted to the first-order sensitivity of the MAC values.

In this paper, a direct method for second-order sensitivity analysis of the MAC value is developed. In Section 2, a brief review is given for common methods of calculating the sensitivity of the MAC value. Section 3 presents the direct method for first-order sensitivity analysis. Following that, the direct method is introduced to calculate the second-order sensitivity of the MAC value by constructing a new Lagrange functional. The effect of normalizations on the proposed method is discussed in Section 5 subsequently. While in Section 6, a simply supported beam and a frame are utilized to validate the accuracy and efficiency of the proposed direct method.

2. Common methods for sensitivity analysis of the MAC value

The eigenvalue problem for undamped symmetric systems can be described as:

$$(-\lambda_i \mathbf{M}+\mathbf{K})\boldsymbol{\varphi}_i=\mathbf{0} \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices, respectively.

The MAC value, which is defined as a scalar constant relating the degree of consistency of different mode shapes, can be given by [1]

$$\mathbf{MAC}_{ij}=\frac{\boldsymbol{\psi}_j^T\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^T\boldsymbol{\psi}_j}{(\boldsymbol{\psi}_j^T\boldsymbol{\psi}_j)(\boldsymbol{\varphi}_i^T\boldsymbol{\varphi}_i)} \tag{2}$$

where $\boldsymbol{\psi}_j$ is the jth measured model shape and $\boldsymbol{\varphi}_i$ is the ith analytical mode shape. $\boldsymbol{\psi}_j^T$ and $\boldsymbol{\varphi}_i^T$ are the transposition of $\boldsymbol{\psi}_j$ and $\boldsymbol{\varphi}_i$, respectively.

In view of Eq. (2), the first-order sensitivity of the MAC value with respect to the design variable p_k can be obtained as [35]

$$\frac{d\mathbf{MAC}_{ij}}{dp_k}=\left(\frac{\partial\mathbf{MAC}_{ij}}{\partial\boldsymbol{\varphi}_i}\right)^T\frac{\partial\boldsymbol{\varphi}_i}{\partial p_k} \tag{3}$$

where the first-order sensitivity of the MAC value with respect to mode shape $\boldsymbol{\varphi}_i$ can be given by

$$\frac{\partial\mathbf{MAC}_{ij}}{\partial\boldsymbol{\varphi}_i}=\frac{2\left(\boldsymbol{\varphi}_i^T\boldsymbol{\psi}_j\right)\boldsymbol{\psi}_j}{\left(\boldsymbol{\psi}_j^T\boldsymbol{\psi}_j\right)(\boldsymbol{\varphi}_i^T\boldsymbol{\varphi}_i)}-\frac{2\left(\boldsymbol{\psi}_j^T\boldsymbol{\varphi}_i\right)\left(\boldsymbol{\varphi}_i^T\boldsymbol{\psi}_j\right)\boldsymbol{\varphi}_i}{\left(\boldsymbol{\psi}_j^T\boldsymbol{\psi}_j\right)(\boldsymbol{\varphi}_i^T\boldsymbol{\varphi}_i)^2} \tag{4}$$

Using the chain rule of differentiation and matrix calculus notations, the second-order sensitivity of the MAC value with respect to the design variable p_k and p_m can then be calculated from Eq. (3), which can be written as [36]

$$\frac{\partial^2\mathbf{MAC}_{ij}}{\partial p_k\partial p_m}=\frac{\partial\boldsymbol{\varphi}_i^T}{\partial p_k}\frac{\partial(\partial\mathbf{MAC}_{ij}/\partial\boldsymbol{\varphi}_i)}{\partial\boldsymbol{\varphi}_i}\frac{\partial\boldsymbol{\varphi}_i}{\partial p_m}+\frac{\partial\mathbf{MAC}_{ij}}{\partial\boldsymbol{\varphi}_i^T}\frac{\partial^2\boldsymbol{\varphi}_i}{\partial p_k\partial p_m} \tag{5}$$

where the second-order sensitivity of MAC value with respect to mode shape $\boldsymbol{\varphi}_i$ can be expressed as

$$\frac{\partial(\partial\mathbf{MAC}_{ij}/\partial\boldsymbol{\varphi}_i)}{\partial\boldsymbol{\varphi}_i}=\frac{2\boldsymbol{\psi}_j\boldsymbol{\psi}_j^T}{\left(\boldsymbol{\psi}_j^T\boldsymbol{\psi}_j\right)(\boldsymbol{\varphi}_i^T\boldsymbol{\varphi}_i)}-\frac{2\left(\boldsymbol{\varphi}_i^T\boldsymbol{\psi}_j\right)\left(2\boldsymbol{\psi}_j\boldsymbol{\varphi}_i^T+2\boldsymbol{\varphi}_i\boldsymbol{\psi}_j^T+\left(\boldsymbol{\psi}_j^T\boldsymbol{\varphi}_i\right)\mathbf{E}\right)}{\left(\boldsymbol{\psi}_j^T\boldsymbol{\psi}_j\right)(\boldsymbol{\varphi}_i^T\boldsymbol{\varphi}_i)^2}+\frac{8\left(\boldsymbol{\psi}_j^T\boldsymbol{\varphi}_i\right)\left(\boldsymbol{\varphi}_i^T\boldsymbol{\psi}_j\right)\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^T}{\left(\boldsymbol{\psi}_j^T\boldsymbol{\psi}_j\right)(\boldsymbol{\varphi}_i^T\boldsymbol{\varphi}_i)^3} \tag{6}$$

Here $\mathbf{E}$ is the unit matrix with the dimension of $n\times n$.

It can be seen that both the first- and second-order sensitivities of the MAC values are depended on the sensitivities of the mode shape. Once the derivatives of mode shape (eigenvector) are calculated, the derivatives of the MAC value can be computed by substituting the eigenvector sensitivities into Eq. (3) or Eq. (5). This method is known as the indirect method. However, *only* after the sensitivities of mode shape with respect to *all* design parameters are calculated *can* the derivatives of MAC_{ij} with respected to those design parameters could be obtained. This indirect method is somewhat computationally expensive [32,35] if the exact methods (the algebraic method and Nelson' s method) are used to calculate these eigen-sensitivites. In the following sections, a direct method for sensitivity analysis of the MAC values will be presented. The direct method can be efficient in computational time and storage capacity when the considered number of design variables is larger than one.

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