Mechanical Systems and Signal Processing 76-77 (2016) 612–633

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Measurement of nonlinear normal modes using multi-harmonic stepped force appropriation and free decay

David A. Ehrhardt*, Matthew S. Allen

Department of Engineering Physics, University of Wisconsin-Madison, 534 Engineering Research Building, 1500 Engineering Drive, Madison, WI 53706, United States

ARTICLE INFO

Article history:
Received 5 June 2015
Received in revised form
14 December 2015
Accepted 27 February 2016
Available online 14 March 2016

Keywords:
Nonlinear normal mode
Experimental force appropriation
Continuous-scan laser Doppler vibrometry

ABSTRACT

Nonlinear Normal Modes (NNMs) offer tremendous insight into the dynamic behavior of a nonlinear system, extending many concepts that are familiar in linear modal analysis. Hence there is interest in developing methods to experimentally and numerically determine a system's NNMs for model updating or simply to characterize its dynamic response. Previous experimental work has shown that a mono-harmonic excitation can be used to isolate a system's dynamic response in the neighborhood of a NNM along the main backbones of a system. This work shows that a multi-harmonic excitation is needed to isolate a NNM when well separated linear modes of a structure couple to produce an internal resonance. It is shown that one can tune the multiple harmonics of the input excitation using a plot of the input force versus the response velocity until the area enclosed by the force–velocity curve is minimized. Once an appropriated NNM is measured, one can increase the force level and retune the frequency to obtain a NNM at a higher amplitude or remove the excitation and measure the structure's decay down a NNM backbone. This work explores both methods using simulations and measurements of a nominally-flat clamped–clamped beam excited at a single point with a magnetic force. Numerical simulations are used to validate the method in a well defined environment and to provide comparison with the experimentally measured NNMs. The experimental results seem to produce a good estimate of two NNMs along their backbone and part of an internal resonance branch. Full-field measurements are then used to further explore the couplings between the underlying linear modes along the identified NNMs.

1. Introduction

Over the past several decades a suite of testing and modeling techniques has been developed to quantify the dynamic motion of a structure using its linear modal properties (e.g. resonant frequencies, mode shapes and damping ratios) [1–3]. The measured or calculated modal properties are typically used to quantify the inertial (mass), elastic (stiffness) and dissipation (damping) characteristics of a structure. Modal properties provide the benefit of describing the global dynamics of a structure using a small number of parameters, and can also be used to predict the response due to an arbitrary excitation thanks to invariance, superposition, etc.. The classical (undamped) Linear Normal Modes (LNMs), that are of interest to this work, can be considered a special case of *Complex Modes*; LNMs are directly related to resonant responses of a structure only

* Corresponding author.
E-mail addresses: dehrhardt@wisc.edu (D.A. Ehrhardt), matt.allen@wisc.edu (M.S. Allen).

http://dx.doi.org/10.1016/j.ymssp.2016.02.063

when dissipation is well approximated as proportional to the mass and stiffness matrices [4]. LNMs are particularly useful in decoupling the equations of motion for such structures, allowing one to find the response of the structure through a summation of individual single degree of freedom responses. Further details can be found in many textbooks [5–8].

When a structure exhibits the nonlinear force–displacement relationships that are typical of geometrically nonlinear structures, the dynamic response characteristics change with amplitude. For large response amplitudes, LNMs no longer uncouple the undamped equations of motion [9] and so the dynamic motion can no longer be thought of as a summation of uncoupled single degree of freedom systems. An alternate definition of modes has been sought, presumably with the hope that an extended superposition principle could be developed and used to solve the nonlinear equations of motion. While that has proved elusive – superposition does not hold for any of the existing definitions of NNMs – these definitions have led to considerable insight into the dynamics of complicated systems and thus proven useful for engineers [10,11]. Rosenberg [12] was the first to pursue a definition, coining the name *Nonlinear Normal Modes* (NNMs) to describe certain *synchronous oscillations* exhibited by the conservative nonlinear equations of motion. This definition was subsequently simplified and expanded to include internal resonances by defining a NNM as simply a *not necessarily synchronous periodic oscillation* of the conservative nonlinear equations of motion [12]. Some NNMs described using this definition originate from a structure's LNMs at low response amplitudes, but NNMs can also describe jumps, bifurcations, internal resonances, modal interactions, sub- and super-harmonic responses which have no counterpart in linear systems [10,11]. Much of the initial work on NNMs to date has focused on analytical or numerical investigations of low-order lumped-mass systems or Galerkin models of simple continua [10–18]. Recently, methods have been developed which extend the numerical calculation of NNMs to more complicated structures which are described by geometrically nonlinear finite element models (FEMs) [19,20].

While NNMs are a new concept, they have found some use already in several applications. For example, NNMs have been used to provide insights to guide the design of nonlinear vibration absorbers [21]. With the extension mentioned above and in [19,20], they are now also being used to characterize FE models of complicated, geometrically nonlinear structures. For example, they were used to correlate simulations [22] with experimental measurements [23,24] and to assess the fidelity of simulation models [20]. The insights that NNMs provide could also be used to guide the design of a nonlinear structure in general or to evaluate advanced model reduction techniques.

This work focuses on an extension of experimental modal analysis (EMA) techniques in which a harmonic forcing is used to experimentally isolate NNMs [22]. EMA methodologies can be broadly split into two areas classified by the number of LNMs measured during a test. Phase Separation Methods (PSMs) excite multiple LNMs at a time with broadband excitation and the response is decomposed using modal parameter estimation techniques to identify the LNMs that are active in the response. These are the most popular and commonly used methods for linear experimental modal parameter identification. Several works have sought to extend these methods to nonlinear identification [9,25,26], but in the presence of nonlinearity the response is not a simple superposition of LNMs. Indeed one may not even be able to accurately postulate the form of the mathematical model a priori, so this becomes challenging. Furthermore, as these methods are applied to higher order systems the number of terms to be identified can become excessive. On the other hand, Phase Resonance Methods (PRMs) excite one LNM at a time by tuning the input force and monitoring the phase of the response until a criterion is met and the measured response becomes the LNM of interest. They are less popular in linear EMA because they can be time-consuming. However, PRMs are still used in Ground Vibration Tests where closely spaced modes occur and highly accurate results (especially in regards to damping) are needed. [27–29]. For linear systems, several methods have been developed [30–33] to identify the distribution of the input force needed to cause the response displacement to lag the input force by 90 degrees, thus fulfilling the phase lag criterion. In the presence of nonlinearity, linear PRMs have been applied so that when the classical phase condition is met, the response is thought to provide a quasi-linearization of the nonlinear structure that can be used to access the dependency of resonant frequencies, structural damping and generalized mass with the amplitude of structural responses [29].

PRMs have also begun to be extended to the measurement of NNMs through the implementation of different methods of force appropriation. For instance, a method called force appropriation of nonlinear systems (FANS) was presented in [34] that uses a multi-point, multi-harmonic force to isolate a single LNM in nonlinear response regimes. This is done by iteratively canceling any coupling between LNMs. Then one can identify the nonlinear characteristics of the isolated mode without modal coupling terms. This work follows a different approach, pioneered by Peeters et al. who presented a rigorous theory by which an undamped NNM could be isolated in a measurement [22,23]. They showed that a multi-point multi-harmonic sine wave could cancel damping and isolate a general NNM. It was then demonstrated both analytically and experimentally that a single-point single harmonic force could be used to isolate a response in the neighborhood of a single NNM with good accuracy [23,24]. In these investigations, once phase lag quadrature was met, the input force was turned off and the response allowed to decay tracing the backbone of the NNM.

This work explores a means whereby a single point multi-harmonic excitation (i.e. a signal containing a fundamental frequency, ω, and various integer harmonics, 2ω, 3ω, 4ω, etc…) can be used to isolate a NNM that are not significantly coupled by damping. The theory shows that a simple metric based on a plot of the input force versus response velocity can be used to monitor the phase condition in real time, and to understand which harmonics should be added to the input force to minimize the phase condition. The results also show that the phases of the response and its harmonics are highly sensitive to perturbations and so the mode indicator function appears to be a more useful metric in gauging whether an adequate estimate of the NNM has been obtained. The methods are tested by measuring two NNMs of a flat