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A combined dynamic analysis method for geometrically nonlinear vibration isolators with elastic rings

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ABSTRACT

A combined analysis method is developed in the present paper for studying the dynamic properties of a type of geometrically nonlinear vibration isolator, which is composed of push–pull configuration rings. This method combines the geometrically nonlinear theory of curved beams and the Harmonic Balance Method to overcome the difficulty in calculating the vibration and vibration transmissibility under large deformations of the ring structure. Using the proposed method, nonlinear dynamic behaviors of this isolator, such as the lock situation due to the coulomb damping and the usual jump resulting from the nonlinear stiffness, can be investigated. Numerical solutions based on the primary harmonic balance are first verified by direct integration results. Then, the whole procedure of this combined analysis method is demonstrated and validated by slowly sinusoidal sweeping experiments with different amplitudes of the base excitation. Both numerical and experimental results indicate that this type of isolator behaves as a hardening spring with increasing amplitude of the base excitation, which makes it suitable for isolating both steady-state vibrations and transient shocks.

1. Introduction

For protecting the isolated structure and the vibration isolator itself from damage, an isolator should isolate both steady-state vibrations and transient shocks. Compared with a linear isolator, a nonlinear isolator could satisfy these requirements simultaneously due to its unique characteristics. Thus, studies on nonlinear dynamic properties and designs of nonlinear isolators are important for practical applications.

There are numerous studies on different nonlinear passive isolators, which have been summarized comprehensively by Ibrahim [1]. This review paper reported that a type of structure based on circular springs has the available nonlinear mechanical characteristics for the vibration isolator. Static mechanical behaviors of the circular springs have been studied by many researchers [2–6]. Tse et al. utilized the equivalent flexural rigidity approach with elliptic integrals to study the static characteristics of composite circular springs under the uniaxial compression [4] and tension [5]. It is found that a single ring performs like a softening spring in the compression region and a hardening spring in the tensile region. Besides, a pair of elastic circular springs in the push–pull configuration proposed in Ref. [6] has the symmetrical hardening stiffness under static compression and tension. Thus, the isolator formed by push–pull configuration rings has potential to satisfy the dual isolation requirements of both steady-state vibrations and transient shocks. Nevertheless, the analytical method proposed

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by Tse et al. [4–6] is fairly complicated and there are currently no convenient and effective methods available for studying the vibration properties of such a geometrically nonlinear isolator with push–pull configuration rings.

If the deformation of the circular springs is large, the problem can be solved numerically with the theory of geometrical nonlinearity on the curved beam. Based on this theory, many researches on calculating large deformations of curved beam structures have been done [7–11]. With the consideration of axial extensibility, Li et al. [7,8] established the exact but complicated ordinary differential governing equations for studying the thermal buckling of an elastic rod with pinned-fixed ends [7] and an Euler-Bernoulli beam supported on a nonlinear foundation [8]. In these studies, the analysis of large static deformations was transformed into the initial-value problem and solved numerically by the shooting method. However, due to the coupling between the axial elongation and the bending deformation, it was difficult to apply this method to solve vibration problems. In addition, with the assumption of axial inextensibility, the formulation and the solution of the dynamic equations of curved beams can be simplified. Santillan et al. [9–11] established partial differential equations for the pinched loop and the upright cantilever beam, both of which were comprised of a thin, inextensible and elastic strip. Since the shooting method is rarely applied to solve the non-linearized partial differential governing equations, two approaches were developed. One is to adopt the assumption of small vibrations, i.e. the linearization for the governing equations [9–10]. The characteristics of in-plane free vibration were studied numerically with the shooting method [9]. The performance of the small-amplitude vibration isolation with this pinched loop was studied by Virgin et al. [10] and the calculated linear displacement transmissibility was verified experimentally. The other approach is to apply the finite difference in time for the corresponding dynamic equations to determine vibration shapes and corresponding frequencies of the beam structure, and then use the perturbation method to find the approximate solutions of dynamic responses [11]. Nevertheless, for the nonlinear isolator with push–pull configuration rings, the first approach fails to calculate the nonlinear dynamic responses under large-amplitude deformations. As for the second approach, there is no need to analyze vibration shapes of the elastic rings themselves because the concerned output responses directly depend on the junction in the configuration and the isolator performs as a single degree-of-freedom system as a whole [6]. In general, there are few studies on the large-amplitude vibrations of curved beams by using the geometrically nonlinear theory directly. In spite of this, due to its effectiveness in calculating large static deformations and the structural feature of the nonlinear isolator, it is reasonable and achievable to transform such a geometrically nonlinear isolator into an equivalent model with lumped parameters.

If a static large deformation of circular rings is obtained, the dynamic properties of such a geometrically nonlinear isolator can be analyzed directly using approximate analytical methods. There are many different approximate analytical methods developed for the analysis of nonlinear vibrations, and the Harmonic Balance Method (HBM) has been considered the simplest and most direct approach to solving the frequency responses of nonlinear systems [12]. Moreover, it is considered to be a powerful method capable of handling strongly nonlinear behaviors and can always converge to an accurate periodic solution for smooth nonlinear systems [13]. The HBM has been extensively applied and promoted by many researchers [14–19]. Ravindra and Mallik [14,15] used the HBM to investigate theoretically a hard Duffing-type isolator under both harmonic base and force excitations. The linear stability analysis and the effects of viscous and coulomb damping were presented. Brennan et al. [16] utilized the HBM to analyze the jump-up and jump-down frequencies of a Duffing oscillator with lightly linear damping. Wang and Zheng [17] proposed a five-parameter polynomial model of wire mesh isolators and identified parameters by the sine-sweep test based on the acceleration transmissibility formulated with the HBM. In addition, the nonlinear isolator with high-static-low-dynamic-stiffness (HSLDS) [18–21] has been widely investigated with the HBM. A. Carrella and M.I. Friswell [18] designed an isolator with HSLDS by exploiting a composite bistable plate and used a hardening HSLDS spring to reduce the whirling of a rotor [19]. Sun et al. [20] studied a 3D quasi-zero-stiffness isolator by the theoretical analysis based on the HBM. Huang et al. [21] employed the Euler buckled beams as a negative stiffness corrector to obtain a HSLDS isolator and investigated its vibration transmissibility theoretically and experimentally. Nevertheless, its discrete sinusoidal test was conducted in an open loop and it resulted in marked differences between the theoretical and experimental results under larger excitation levels. In general, the above papers applied the HBM to study the effect of coulomb damping [15], the jump phenomenon [14,16], the identification for parameters in the isolator [17] and the nonlinear dynamic analysis of the HSLDS [18–21], which provided constructive guidance for studying the geometrically nonlinear isolator with elastic rings. Therefore, the HBM will be appropriate and competent for analyzing the dynamics of such a geometrically nonlinear isolator.

In the present paper, a simple and effective dynamic analysis method for studying the geometrically nonlinear vibration with large deformations is proposed, which is extremely beneficial for the design of the geometrically nonlinear isolator in practical applications. In this approach, the geometrically nonlinear theory of curved beams and the HBM are combined to avoid the linearization assumption with small vibrations [9–10] and the relatively complicated finite difference in time [11]. For introducing the method and showing the application procedure, this combined method will be applied to study the vibration properties of the isolator constituted by two pairs of elastic rings in the push–pull configuration with the viscous, coulomb and quadratic damping. A flowchart of the proposed method is shown in Fig. 1, where the red lines denote the demonstrations from numerical simulations and vibration experiments. In this study, the geometrically nonlinear theory of curved beams is first applied to model the deformation of the elastic rings in the push–pull configuration and the shooting method is used to numerically solve the equations. Then, an odd polynomial function is employed to fit the numerical results for obtaining the relationship between the restoring force and the deformation. Lastly, with the polynomial function, the HBM is applied to establish and solve the nonlinear dynamic equation of the isolator. Numerical results on the relative displacement and the absolute acceleration transmissibility obtained with the proposed method are compared with the direct numerical integration. In addition, slowly sinusoidal sweeping experiments controlled in the close loop with a shaker table are conducted to further validate the proposed method.

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