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The spectral analysis of cyclo-non-stationary signals

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ABSTRACT

Condition monitoring of rotating machines in speed-varying conditions remains a challenging task and an active field of research. Specifically, the produced vibrations belong to a particular class of non-stationary signals called cyclo-non-stationary: although highly non-stationary, they contain hidden periodicities related to the shaft angle; the phenomenon of long term modulations is what makes them different from cyclostationary signals which are encountered under constant speed regimes. In this paper, it is shown that the optimal way of describing cyclo-non-stationary signals is jointly in the time and the angular domains. While the first domain describes the waveform characteristics related to the system dynamics, the second one reveals existing periodicities linked to the system kinematics. Therefore, a specific class of signals – coined angle-time cyclostationary is considered, expressing the angle-time interaction. Accordingly, the related spectral representations, the order-frequency spectral correlation and coherence functions are proposed and their efficiency is demonstrated on two industrial cases.

1. Introduction

In the last decade, the cyclostationary theory has played a pivotal role in vibration-based condition monitoring. In particular, it has helped to improve the diagnosis of roller bearings and gears. When it comes to characterizing cyclostationary vibration signals, the spectral correlation is one of the most powerful second-order tool that completely describes the waveform dynamics and the nature of hidden periodicities. Randall et al. [2] were the first who demonstrate its efficiency for rolling element bearing diagnostics. More analytical studies on the same subject together with related estimation issues are found in Refs. [4] and [5]. However, the spectral correlation and its statistical characteristics were all conceived under the assumption of stationary or quasi-stationary regimes wherein the speed profile remains merely constant.

In real applications, however, rotating machines are sometimes subjected to large speed variations which jeopardize the stationary regime assumption. Some typical examples are provided by wind turbines or engines during a run-up: in the former case speed variations are endured and in the latter case they are produced on purpose. The corresponding signals are no longer cyclostationary (CS), although they are still exhibiting rhythms produced by some cyclic phenomena. A typical example is the vibration signal produced by a series of (deterministic) impacts locked to some rotating components during a

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Notations

AT	angle–time
AT-CS	angle–time cyclostationary
CS	cyclostationary
CS1	first-order cyclostationary
CS2	second-order cyclostationary
SC	spectral correlation
SCoh	spectral coherence
OFSC	order-frequency spectral correlation
OFSCoh	order-frequency spectral coherence
$\mathbb{E}$	Ensemble averaging operator
$\mathcal{F}_{\tau\to f}$	Fourier transform with respect to time lag
$\mathcal{F}_{t\to\alpha_t}$	Fourier transform with respect to time
$\mathcal{F}_{\theta\to\alpha_\theta}$	Fourier transform with respect to angle
t	time variable
θ	angle variable
τ	time-lag variable
φ	angle-lag dual
f	spectral frequency (time-lag dual)
α_t	cyclic frequency (time dual)
α_θ	cyclic order (angle dual)
BPOO	ball pass order on the outer race
BPOI	ball pass order on the inner race

variable speed regime. Obviously, the signal would not be periodic in time due to varying spacing between the impacts. Nevertheless, the spacing of the impacts would be constant in angle – hence defining the notion of "cycle" – but the resulting signal would still not be periodic in angle due to two reasons: first, the response to each impact has constant time characteristics such as resonance frequency and relaxation time, which would become varying in angle; second, the strengths of the impacts are likely to be modulated by the regime. This latter aspect (amplitude modulation caused by speed variations) was investigated in Refs. [8] and [6]. In such cases, the CS framework is generally insufficient to describe and analyze such signals. A similar issue has been encountered in the field of telecommunication [9]. For instance, it has been shown that a signal subjected to a Doppler shifts loses its CS properties due to the relative motion between transmitter and receiver. The novel class of "generalized almost-cyclostationary" processes has been introduced to embody this particular case [10–12]. Despite its relevance in the mentioned field, this generalization is not able to deal with the mechanical signals of interest in this paper.

In the last few years, efforts have been directed toward the extension of existing CS tools in nonstationary regimes. In particular, D'Elia et al. [18] were the first to explore the order-frequency approach. Their idea was to replace the frequency–frequency distribution by a frequency-order distribution that jointly describes the time dynamics and the angle periodicities of the signal. They proposed intuitive estimators to extent the SC and *cyclic modulation spectrum* which were coined as the "α-synchronized spectral correlation density" and "α-synchronized cyclic modulation spectrum", respectively. Later on, Urbanek et al. [26] proposed an angle–frequency distribution – namely the *averaged instantaneous power spectrum* – based on a time filtering step followed by an angle averaging operation of the squared output. A similar solution was proposed by Jabłoński et al. [27] who introduced the *angular-temporal spectrum* to jointly represent the angular and temporal properties of the signal. Other solutions based on the order spectrum of the squared envelope after some preprocessing steps were introduced in Refs. [13–17]. However, all these attempts still lack a formalism with rigorous statistical definitions. The aim of this paper is then to partially fill in this gap by considering an angle/time cyclostationary (AT-CS) approach.

This paper is organized as follows: Section 2 is concerned with a brief review of stationary, cyclostationary, and cyclo-non-stationary signals as well as the spectral correlation and the spectral coherence. Section 3 introduces the order-frequency spectral correlation function (OFSC) and the order-frequency spectral coherence function (OFSCoh) from a joint angle–time vision. Next, Section 4 deals with estimation issues by proposing a consistent Welch-based estimator and related statistical thresholds. Eventually, the proposed tools are illustrated in Section 5 on real data from test rigs. A first application is focused with bearing diagnostics and a second one is on the exploitation of these tools to detect gear rattle noise in vibration signals recorded on an automotive gearbox in run-up conditions.

2. Problem statement

2.1. Stationary, cyclostationary, and cyclo-non-stationary signals

The notion of stationary, cyclostationary and cyclo-non-stationary signals is central to this paper; this first subsection provides a brief summary of their definitions with emphasis on the first (mean value) and second orders (covariance functions). Although not general, it often happens that the first two orders are enough to cover many practical purposes in mechanical engineering applications.

2.1.1. Stationary signals

Stationary signals are those with constant statistics along the time axis. Formally speaking, a signal $x(t)$ is said first-order stationary if its mean (i.e. its first-order moment) does not depend on time t, i.e.

$$M_{1x} = \mathbb{E}\{x(t)\}, \quad for\ all\ t, \tag{1}$$

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