Automatica 54 (2015) 348–359

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Filtering and fault detection for nonlinear systems with polynomial approximation[☆]

Yang Liu[a], Zidong Wang[b], Xiao He[a], D.H. Zhou[a,1]

[a] Department of Automation, TNList, Tsinghua University, Beijing 100084, PR China

[b] Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom

ARTICLE INFO

Article history:
Received 12 January 2014
Received in revised form
4 December 2014
Accepted 4 February 2015
Available online 4 March 2015

Keywords:
Polynomial filtering
Nonlinear systems
Polynomial approximation
Fault detection
Adaptive threshold
Recursive algorithm
Kronecker power

ABSTRACT

This paper is concerned with polynomial filtering and fault detection problems for a class of nonlinear systems subject to additive noises and faults. The nonlinear functions are approximated with polynomials of a chosen degree. Different from the traditional methods, the approximation errors are not discarded but formulated as low-order polynomial terms with norm-bounded coefficients. The aim of the filtering problem is to design a least squares filter for the formulated nonlinear system with uncertain polynomials, and an upper bound of the filtering error covariance is found and subsequently minimized at each time step. The desired filter gain is obtained by recursively solving a set of Riccati-like matrix equations, and the filter design algorithm is therefore applicable for online computation. Based on the established filter design scheme, the fault detection problem is further investigated where the main focus is on the determination of the threshold on the residual. Due to the nonlinear and time-varying nature of the system under consideration, a novel threshold is determined that accounts for the noise intensity and the approximation errors, and sufficient conditions are established to guarantee the fault detectability for the proposed fault detection scheme. Comparative simulations are exploited to illustrate that the proposed filtering strategy achieves better estimation accuracy than the conventional polynomial extended Kalman filtering approach. The effectiveness of the associated fault detection scheme is also demonstrated.

1. Introduction

The past few decades have seen the nonlinear state estimation problem as a recurring research theme due to the pervasive existence of nonlinearities in almost all real-world industrial systems. If not adequately dealt with, the intrinsic nonlinearities may lead to undesirable dynamic behaviors such as oscillation or even instability. Indeed, the nonlinear analysis issue has been the main stream of research for systems control and estimation problems attracting researchers from a variety of disciplines. So far, much effort has been devoted to the estimation/filtering problems for nonlinear systems, see e.g. Basin, Elvira-Ceja, and Sanchez (2012), Basin, Shi, and Calderon-Alvarez (2009), Caballero-Águila, Hermoso-Carazo, and Linares-Pérez (2012), Dong, Wang, and Gao (2013), Gao and Chen (2007), Gershon, Shaked, and Yaesh (2005), Hermoso-Carazo and Linares-Pérez (2007), Hu, Wang, Shen, and Gao (2013), Karimi (2009), Li, Lam, and Chesi (2012), Shu, Lam, and Xiong (2009), Wei, Wang, and Shu (2009), and the references therein. Among others, the renowned extended Kalman filter (EKF) algorithm has proved to be an effective method to solve the estimation problem for nonlinear systems in the least mean square sense. Recently, considerable attention has been paid to the performance improvement of the traditional EKF with respect to the insensitivity to the parameter uncertainties as well as the capability of handling nonlinearities (James & Peterson, 1998; Reif, Gunther, Yaz, & Unbehauen, 1999; Xiong, Liu, & Liu, 2011). When the system states and observations are polynomial with additive Gaussian white noises, the mean-square filter has been designed where the statistical characteristics of Gaussian distribution have been made use of to recursively calculate the filtering error covariance (Basin, 2008; Basin & Rodriguez-Ramirez, 2012; Basin, Shi, & Calderon-Alvarez, 2010).

☆ This work was supported by the National Natural Science Foundation of China under Grants 61490701, 61210012, 61290324, 61473163, and 61273156, and Jiangsu Provincial Key Laboratory of E-business at Nanjing University of Finance and Economics of China under Grant JSEB201301. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Michael V. Basin under the direction of Editor Ian R. Petersen.

E-mail addresses: liuyang-10@mails.tsinghua.edu.cn (Y. Liu), Zidong.Wang@brunel.ac.uk (Z. Wang), hexiao@tsinghua.edu.cn (X. He), zdh@mail.tsinghua.edu.cn (D.H. Zhou).

[1] Tel.: +86 10 62794461; fax: +86 10 62786911.

http://dx.doi.org/10.1016/j.automatica.2015.02.022
0005-1098/

Polynomial extended Kalman filter (PEKF) is an extension of EKF with aim to cater for inherent nonlinearities using polynomial approximations. Traditional EKF is only concerned with the *linear* term and simply ignores the linearization error, while PEKF considers the Carleman approximation of a nonlinear system of a given order μ (Kowalski & Steeb, 1991). The order could be determined according to the form of the nonlinearity and the estimation performance specification. In this sense, the PEKF is more applicable than EKF as far as the accuracy is concerned. When the order $\mu = 1$, PEKF reduces to conventional EKF. A PEKF is designed to cope with an augmented state which is made of Kronecker powers of the original state (Germani, Manes, & Palumbo, 2005). Due to its higher accuracy than that of EKF, the PEKF has stirred quite a lot research attention and many corresponding results have been reported in the literature (Germani, Manes, & Palumbo, 2003, 2007, 2009; Mavelli & Palumbo, 2010). Nevertheless, the PEKF approach still ignores the Carleman approximation errors which would give rise to certain conservatism especially when the nonlinearities are severe.

In theory, well-behaved nonlinear functions could only be approximated *accurately* by polynomials whose orders approach *infinity*. In engineering systems, however, the polynomials with extremely high orders are difficult to be realized. A feasible way is to determine the finite order of the polynomials for satisfactory approximation of the nonlinear dynamics according to the degree of the nonlinearities and the nature of the research problem. As such, the unavoidable high-order approximation errors would be impacting on the estimation performances that should be taken into account. While EKF and PEKF work quite well for system with relatively low degree of nonlinearities, the classical EKF algorithm ignores the linearization errors and most available PEKF approaches discard the Carleman approximation errors. It is noted that the approximation errors differ greatly from each other due to various forms of nonlinearities. Instead of being simply dropped, the approximation errors do offer further room for improving the estimation accuracy if properly coped with. This seemingly natural idea, unfortunately, would inevitably bring us substantial challenges when calculating the covariances of the estimation errors in the least mean square sense since the approximation errors could not be exactly known. Moreover, coupled with both the low-order terms and external disturbances, the approximate errors would become very complicated to analyze. These identified difficulties motivate us to address the PEKF problem *by allowing for the Carleman approximation error* with aim to obtain higher approximation accuracy and better estimation performance.

The fault detection (FD) problem is another active research topic in control engineering due primarily to the increasingly higher and higher safety requirements. Yet, the FD problem provides an ideal platform to demonstrate the applicability of the polynomial filter technique to be developed. Since a properly designed filter could generate residual signal so as to efficiently detect abnormal changes in the system, filter/observer based FD has become a common technique. In filter/observer based FD methods, a fault would be detected and diagnosed effectively via comparing the actual system output with the filter output signal since the faults would normally bring in unexpected variations in the system measurement. To date, a great number of results have been published on this issue, see Zhou, He, Wang, Liu, and Ji (2012), He, Wang, Liu, and Zhou (2013), Shen, Wang, and Ding (2013), Zhang, Jiang, and Shi (2012) and the references therein. Residual evaluation, which consists of threshold and decision logic, is critical to the performance of FD. For systems varying with time or parameters, the *adaptive threshold* has been of particular research importance for its better trackability and faster self-adjustment as compared with the constant threshold.

So far, many existing results have focused on adaptive threshold generation for linear systems (Meseguer, Puig, Escobet, & Saludes, 2010; Montes de Oca, Puig, & Blesa, 2012; Puig, Quevedo, Escobet, Nejjari, & de las Heras, 2008; Zhong, Ding, Lam, & Wang, 2003). However, the corresponding results for nonlinear systems have been scattered in spite of their engineering significance (Khan & Ding, 2011; Shi, Gu, Lennox, & Ball, 2005). In Zhang, Polycarpou, and Parisini (2001) and Zhang (2011), the nonlinear dynamics have been assumed to be uniformly bounded and the bounds have been utilized to determine the adaptive threshold. In the context of polynomial fault detection for nonlinear systems, the adaptive threshold determination problem is essentially difficult mainly for two reasons: (1) the expressions of the disturbances and approximation errors are sophisticated and their influences on the threshold remain unclear; and (2) it is non-trivial to calculate the bounds of the disturbances and approximation errors in order to propose a reasonable fault detection threshold. Up to date, the polynomial fault detection scheme has not been fully investigated, not to mention the case where the Carleman approximation errors are taken into consideration. This constitutes another motivation of our present work.

Summarizing the discussions made above, in this paper, both the polynomial filtering and fault detection problems are thoroughly investigated for a class of nonlinear systems. The Carleman approximation of a given order is introduced to approximate the nonlinear functions. In contrast to existing literature, the high-order approximation errors are explicitly taken into account in terms of low-order polynomials with uncertain but bounded coefficients. A time-varying filter is first designed at each time step to guarantee that the filtering error covariance is bounded in the fault-free case. Such a bound is subsequently minimized with respect to a properly designed filter gain. To show the applicability of the proposed filter scheme, a fault detection strategy is then proposed consisting of the calculation of adaptive threshold and decision logic. The filter gain and adaptive threshold are determined using the information from the approximation errors and additive disturbances. With the fault detection strategy, the fault detectability is also investigated.

The main contribution of the paper can be highlighted as follows: (1) a polynomial estimation scheme for a class of nonlinear systems is presented by taking account of the Carleman approximation errors, which leads to higher estimation accuracy; (2) an upper bound of the estimation error covariance in the polynomial scheme is calculated and minimized by designing an appropriate filter gain; (3) adaptive fault detection threshold is developed for the desired polynomial filter to realize efficient detection and the fault detectability is analyzed; and (4) the proposed algorithms for both the filter gain design and the adaptive threshold computation are recursive and therefore suitable for online applications. The rest of paper is organized as follows. In Section 2, the Carleman approximation and the approximation error analysis are introduced for nonlinear systems. The polynomial filter design problem is solved in Section 3 and the adaptive fault detection scheme is established in Section 4. A simulation example is illustrated in Section 5 and the paper is concluded in Section 6.

Notations. The notation used in the paper is standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscripts A^T and A^{-1} denote the transpose and inverse of matrix A, respectively. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). I is the identity matrix with compatible dimension. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\|A\|$ denotes the spectral norm of matrix A, and $\|x\|$ refers to the Euclidean norm of vector $x \cdot \otimes$ is the Kronecker product defined as $A \otimes B = \begin{bmatrix} a_{1,1}B & \cdots & a_{1,n}B \\ \vdots & \ddots & \vdots \\ a_{m,1}B & \cdots & a_{m,n}B \end{bmatrix}$. $x^{[m]}$ represents the mth Kronecker power of