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Empirical mode decomposition as a time-varying multirate signal processing system

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ABSTRACT

Empirical mode decomposition (EMD) can adaptively split composite signals into narrow subbands termed intrinsic mode functions (IMFs). Although an analytical expression of IMFs extracted by EMD from signals is introduced in Yang et al. (2013) [1], it is only used for the case of extrema spaced uniformly. In this paper, the EMD algorithm is analyzed from digital signal processing perspective for the case of extrema spaced nonuniformly. Firstly, the extrema extraction is represented by a time-varying extrema decimator. The nonuniform extrema extraction is analyzed through modeling the time-varying extrema decimation at a fixed time point as a time-invariant decimation. Secondly, by using the impulse/summation approach, spline interpolation for knots spaced nonuniformly is shown as two basic operations, time-varying interpolation and filtering by a time-varying spline filter. Thirdly, envelopes of signals are written as the output of the time-varying spline filter. An expression of envelopes of signals in both time and frequency domain is presented. The EMD algorithm is then described as a time-varying multirate signal processing system. Finally, an equation to model IMFs is derived by using a matrix formulation in time domain for the general case of extrema spaced nonuniformly.

1. Introduction

It is well known that non-stationary signals are widely existed in the engineering application. For example, the mechanical faults may be non-stationary and generate transient events [2]. Some methods such as advanced time-frequency analysis techniques are proposed to deal with non-stationary signals. Empirical mode decomposition (EMD), introduced by Huang et al. [3], is just one of the method to deal with nonstationary signals adaptively, so it has attracted much attention. EMD has been extensively studied and widely applied in fault diagnosis of rotating machinery [2], such as aircraft damage detection [4], linear friction weld process monitoring [5], bearing fault diagnosis [6].

EMD can separate a composite signal into a finite number of simple oscillatory mode functions called intrinsic mode functions (IMFs) which represent the oscillation modes imbedded in the data. An IMF which represents the oscillation modes imbedded in the data is identified by two characteristics [3]: 1) the number of extrema and zero-crossings either be equal or differ at most by one, and 2) the upper envelope defined by the local maxima and the lower envelope defined by the local minima are symmetric about the zero line. For a given discrete signal $x(n)$, the process of extracting an IMF can be described as follows [3].

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1) Identify all the local extrema of $x(n)$.
2) Interpolate between maxima (resp., minima) by a cubic spline to form the upper envelope $e^u_{ij}(n)$ (resp., the lower envelope $e^d_{ij}(n)$).
3) Calculate the mean of the envelopes $\bar{e}_{ij}(n) = (e^u_{ij}(n) + e^d_{ij}(n))/2$.
4) Compute $b_{ij}(n) = x(n) - \bar{e}_{ij}(n)$.
5) If $b_{ij}(n)$ is an IMF, then set $c_i(n) = b_{ij}(n)$. Otherwise, iterate the above steps on the signal $b_{ij}(n)$.
6) Compute the residue $r_i(n) = x(n) - \sum_i c_i(n)$.

Steps (1–4) are known as one times sifting. Generally speaking, more than one time sifting iterations are needed to extract an IMF.

However, EMD which is a data-driven method lacks a rigorous mathematical foundation [7–10] though it has a successful application in engineering. A general theoretical framework of the EMD method can provide a useful theoretical and technical support for its application. Although it is a problem difficult to formulate the EMD algorithm, some useful attempt has been done to solve this problem. In [11–13], an alternate procedure that the mean envelope of signals is detected by a parabolic partial differential equation (PDE) based approach is adopted to formulate EMD. The interest of the PDE-based approach is to avoid the extrema interpolation problem because the sifting is very sensitive to the kind of the spline interpolation used. Another way to formulate EMD is by using the multirate signal processing theory. From the digital signal processing point of view, the EMD algorithm can be regarded as a multirate signal processing system [1]. The first step of EMD is a process of extrema sampling, and the second step is a process of signal reconstruction [14,15]. Cubic spline interpolation [3], the PDE-based envelopes detection [11–13], the genetic algorithm-optimized piecewise polynomial interpolation [16], and trigonometric interpolation [17] are all the signal reconstruction methods.

Envelopes of signals are critical to formulate the EMD algorithm. Widely accepted definition of envelopes is associated with analytic signals. The signal $x(n)$ is termed analytic if and only if $\hat{x}(n) = -jx(n)$ where $\hat{x}(n)$ is the Hilbert transform of $x(n)$ [18]. The modulus $|e(n)| = [x^2(n) + \hat{x}^2(n)]^{1/2}$ is termed the envelope of $x(n)$ [19,20]. It would be of more interest to mention that an analytic signal contains no negative frequencies. Furthermore, Boashash [21] points out that the envelope of an actual waveform has physical significance only for narrow-band signals and cannot be measured precisely except for pure sinusoids. Hence, it is difficult to express the EMD algorithm by using the expression of envelopes defined through analytic signals.

It is wonderful that envelopes of signals used in the EMD algorithm are calculated through spline interpolation instead of analytic signals. So, splines are critical to analyze the EMD method. Through analyzing cubic B-spline interpolation, it is shown in [1] that the process of forming the upper and lower envelopes of signals involves three steps: 1) extrema sampling, 2) interpolation, and 3) filtering by a cubic B-spline filter. Although a general analytical expression of IMFs is presented in [1], it is only used for the case of extrema spaced uniformly. It is more general that extrema are spaced nonuniformly for composite signals. In this paper, we aim to provide an expression of the EMD algorithm for the case of extrema spaced nonuniformly.

2. Nonuniform extrema decimation

In the EMD algorithm, the maxima sequence and the minima sequence are extracted independently. From digital signal processing perspective, the process of extracting maxima from signals is an operator of maxima decimation. Similarly, the process of extracting minima from signals is an operator of minima decimation. For simplicity, we only focus on the analysis on the maxima decimation. The analysis results for maxima decimation in this section are also applied to the minima decimation.

Let $x^u(m)$ represent the sequence of maxima of the signal $x(n)$. Suppose the maxima are located at $N^u_m,\ \ N^u_m \in Z$ where Z represents integers. Then we have

$$x^u(m) = x(N^u_m). \tag{1}$$

If we let

$$p^u_m = N^u_m - N^u_{m-1}, \tag{2}$$

and

$$N^u_0 = 0,$$

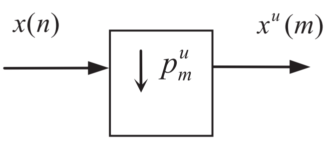

Fig. 1. Block diagram of a nonuniform maxima decimator.