A novel parameter-induced stochastic resonance phenomena in fractional Fourier domain

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ABSTRACT

The parameter-induced stochastic resonance (SR) phenomenon in a novel self-adaptive dynamical system driven by linear frequency modulated (LFM) signal and additive noise is considered from the view of the signal-to-noise ratio (SNR). It is found that the dynamical system can be perfectly analyzed by equivalently transforming it into a traditional first-order linear dynamical system driven by periodic signal and additive noise in fractional Fourier transform (FrFT) domain with an optimal rotated angle, and the theoretical analysis and simulation results show that output SNR exhibits the SR behavior when it is plotted as a function of the system parameter. Furthermore, the optimal value of adjusted parameter is obtained, and the possible area of SNR gain is theoretically determined only by center-frequency and modulated frequency of the driving LFM signal.

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1. Introduction

The concept of stochastic resonance (SR) was originally put forward by Benzi et al. [1]; wherein, they addressed the problem of the periodically recurrent ice ages. Since then, SR has continuously attracted considerable attention in theory and experiment. The term is given to a phenomenon that is manifest in some dynamical systems whereby generally feeble input information can be amplified and optimized by the assistance of noise, and it reveals the construction effect of noise to ordered systems, and changes the traditional viewpoint that noise just has the effect of destruction.

Incipient research focused on nonlinear system driven by periodic signal and noise [2,3]. Then, the theory is extended. Many investigations show that SR also exists in linear system subject to multiplicative colored noise [4,5]. Moreover, the conventional SR means that the signal-to-noise ratio (SNR) versus the noise intensity exhibits a peak [6,7], while the generalized SR, which was introduced by Gitterman et al. [8,9], implied the non-monotonic behaviors of certain functions of the output signal, such as moments and autocorrelation function, on the noise and system parameters. Furthermore, Zhong et al. [10] investigated SR phenomenon in the under damped linear fractional Langevin equation under the external periodic force and multiplicative symmetric dichotomous noise. Recently, Chen et al. [11,12] explore the underlying mechanism of the SR phenomenon respectively by the fixed detector and variable detector. Furthermore, based on SR theory, weak signal detection made great success, particularly in the field of machinery fault diagnosis [13]. And Ref. [14] refined the model to

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achieve the mapping from measured high-frequency signal to a low frequency band, achieving the purpose of the SR detecting weak signal in the context of high frequency.

In all of the above literature, the proposed SR systems are usually restricted to periodic signal. Actually, aperiodic linear frequency modulated (LFM) signal has been widely used in various systems, such as the fractional Fourier transform (FrFT) based orthogonal frequency division multiple access (OFDMA) system [15–21], in which the traditional periodic exponential subcarrier signal bases are replaced by the LFM subcarrier signal bases, the received signals comprise desired LFM signal from target base station (BS), interfering LFM signals from ambient BSs and additive white Gaussian noise (AWGN). The theoretical analysis and numerical simulation results showed that some equalizers [15,18] in optimal FrFT domain improved the system performance, especially in the frequency selective channel [19] or in the presence of carrier frequency offset (CFO) [20], as compared to the traditional Fourier domain method. However, the performance of cell edge users decreased obviously because of the heavy interference from ambient BSs [21]. Another application is the radar system for target detection and localization [22–26].

Although, in 1995, Collins et al. combined SR and information theory, put forward the aperiodic SR theory [27,28], in which the average mutual amount of information, the bit error rate (BER) and the channel capacity, as the means of SR measurement, were used to solve that aperiodic signal. Unfortunately, as to the widely used LFM signal, there was little progress. Recently, Peng et al. [29] proposed the LFM signal driving over-damped bi-stable dynamical system and analyzed its SR behaviors, and Wang et al. [30] proposed the multiple LFM signals driving system to filter the received signal and suppress the inter-cell inference (ICI). Because of the aperiodicity of LFM signal and the distortion motivated by nonlinear system, it was improper to characterize SR within traditional signal to interference-plus-noise ratio (SINR) in frequency domain. Therefore, based on the energy aggregation of LFM signal in optimal FrFT domain [31,32], SINR defined in optimal FrFT domain was proposed to study the system performance, and the simulation results showed coordinative stochastic resonance (CSR) phenomenon with the underlying mechanism that interfering LFM signals assisted weak noise with particle hopping between two wells, coherently to the frequency change of desired LFM signal, and interfering energy was partly transformed into that of the desired signal by the CSR filter. Unfortunately, the statistical properties of the output signal were ignored due to the difficult analysis of nonlinear system, and the system was suitable only for LFM signals with very low center-frequency and modulated frequency. Moreover, the performance gain disappeared with the increasing of instant driving frequency. Actually, based on adiabatic approximation theory [33], traditional SR behavior occurs only in the case of small parameters. Although, under the large-parameter condition, Leng et al. proposed scale transformation SR system [34–36], in which the external periodic driving force with high frequency was conventionally processed by the pre-proceeding method, that is, mapping the parameters into the small parameters with the scale transform factor. Whereas, this existing method was still not suitable for SR systems driven by the aperiodic force.

In this paper, a novel parameter-induced SR dynamical system is proposed as SR filter to suppress the interference, and the performance is theoretically analyzed by equivalently transforming the model into a traditional first-order linear dynamical system driven by the periodic signal and noise in optimal FrFT domain, and the theoretical possible area of SNR gain is obtained. Simulation results show that the output SNR of the dynamical system exhibits the SR behavior when it is plotted as a function of the system parameter, and the optimal parameter, according to the SR peak, can be used to improve the system performance obviously.

2. System model and problem description

2.1. Research background

The structure of FrFT-based OFDM baseband system with SR filter is shown in Fig. 1. At the transmitter, symbol information is modulated by orthogonal LFM base, and assume that the modulated signal propagates through an AWGN channel, the received signal carrying symbol information β_k from the serving BS, at the kth subcarrier, can be written as

$$r_k(t) = s_k(t) + \sqrt{2D}\xi_k(t), \tag{1}$$

where $\sqrt{2D}\xi_k(t)$ represents AWGN with zero mean and power spectrum intensity $2D$, $s_k(t) = a_k\beta_k g_k(t)$ is the desired signal
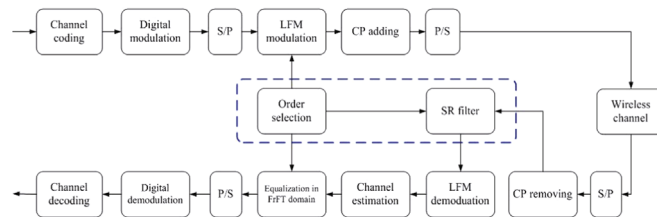

Fig. 1. Block diagram of an FrFT-OFDM system with the optimal order selection and SR filter.