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Brief paper

Decentralized adaptive tracking control for a class of interconnected nonlinear time-varying systems ☆

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ABSTRACT

In this paper, aiming at output tracking, a decentralized adaptive backstepping control scheme is proposed for a class of interconnected nonlinear time-varying systems. By introducing a bound estimation approach and two smooth functions, the obstacle caused by unknown time-varying parameters and unknown interactions is circumvented and all signals of the overall closed-loop system are proved to be globally uniformly bounded, without any restriction on the parameters variation speed. Moreover, it is shown that the tracking errors can converge to predefined arbitrarily small residual sets with prescribed convergence rate and maximum overshoot, independent of the parameters variation speed and the strength of interactions. Simulation results performed on double inverted pendulums are presented to illustrate the effectiveness of the proposed scheme.

1. Introduction

Decentralized adaptive control for uncertain interconnected systems has long been an active issue in the control community for both theoretical and practical importance. In this control strategy, a local controller is designed for each subsystem using only local signals, which, while simplifying the controllers structure, brings challenge to the stability and performance analysis of the overall closed-loop system in the presence of uncertain interactions among subsystems. In the early stage of the research, decentralized adaptive control schemes were developed mainly based on the traditional certainty equivalence principle and some conservative assumptions were usually required on subsystems structure and interactions; see, for instance, Gavel and Siljak (1989), Ioannou (1986), Ioannou and Kokotovic (1985) and Shi and Singh (1992) and the references therein for more details.

Since the 1990s, backstepping has become one of the most popular design techniques for uncertain linear and nonlinear systems (Cai, Wen, Su, & Liu, 2013; Krstic, Kanellakopoulos, & Kokotovic, 1995; Wang & Lin, 2010, 2012), owing to some advantages such as providing a promising way to improve the transient performance. The first decentralized adaptive backstepping control scheme was reported in Wen (1994). In Ye (1999) and Ye (2011), by integrating the backstepping design with Nussbaum functions, decentralized adaptive control strategies were proposed for interconnected nonlinear systems whose control directions are unknown. In Liu and Xie (2010), Wen, Zhou, and Wang (2009) and Zhang, Wen, and Soh (2000), interconnected systems with dynamic interactions were investigated. Introducing a low pass filter at each design step, a simplified adaptive backstepping design was presented in Yoo, Park, and Choi (2009) at the price of sacrificing global stability. However, it is noticed that these schemes (Liu & Xie, 2010; Wen, 1994; Wen et al., 2009; Ye, 1999, 2011; Yoo et al., 2009; Zhang et al., 2000) are only applicable to output regulation. When considering output tracking, the task becomes more complicated because the nonzero desired trajectories to be tracked will affect the dynamics of other subsystems through interactions. Assuming that the interactions are bounded by some polynomials, the output tracking problem was addressed in Jain and Khorrami (1997), and this polynomial-type growth condition was later relaxed in Jiang (2000). Nevertheless, the result obtained in Jiang (2000) is only partially decentralized since information exchange among subsystems is required. In Zhou and Wen (2008), a totally decentralized adaptive output tracking control scheme was proposed via backstepping design and the root-mean-square performance of the tracking errors was guaranteed. However, this type of performance

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cannot prevent some undesired transient behaviors such as bursting (Zhang & Ioannou, 2000).

In spite of the progress, it should be pointed out that, in all aforementioned decentralized adaptive control schemes, the unknown parameters of each subsystem are limited to be constants. In the more general case of unknown time-varying parameters, particularly when the control coefficients are time-varying, these schemes are hard to be applied. In fact, even in the single-input single-output (SISO) case, the design of stable adaptive controllers for time-varying systems is a challenge and only a few results are available in the literature, because it is hard to estimate unknown parameters which vary with time and their nonzero derivatives may significantly complicate the problem. In Fidan, Zhang, and Ioannou (2005) and Zhang, Fidan, and Ioannou (2003), adaptive backstepping control methods were developed for SISO linear time-varying systems, which, under the restriction that the unknown parameters vary slowly, are able to steer the tracking error to be of the order of parameters variation speed in the mean square sense. In Ge and Wang (2003), a class of SISO nonlinear systems whose parameters change within known closed intervals was investigated using backstepping design, but the transient performance analysis and the extension to interconnected systems were not considered.

In this paper, aiming at output tracking, a novel decentralized adaptive backstepping control scheme is proposed for a class of interconnected nonlinear time-varying systems, which possesses the following features:

- The systems considered are pure time-varying and interconnected in the sense that all parameters are allowed to vary with time and exist together with the interactions in every equation of each subsystem. With the aid of two smooth functions incorporated into the controllers design, the effect of the unknown time-varying parameters and the unknown interactions is successfully counteracted and global stability of the overall closed-loop system is obtained.
- Instead of directly estimating the unknown time-varying parameters, we estimate their bounds and no effort is needed to deal with their derivatives. As a result, the system parameters are only required to be bounded and piecewise continuous, without any restriction on the variation speed.
- By employing the prescribed performance control technique proposed in Bechlioulis and Rovithakis (2009), Kostarigka, Doulgeri, and Rovithakis (2013) and Wang and Wen (2010), it is shown that all tracking errors can converge to predefined arbitrarily small residual sets with prescribed convergence rate and maximum overshoot, independent of the parameters variation speed and the strength of interactions.

The remainder of this paper is organized as follows. In Section 2, the control problem is introduced. Section 3 gives the decentralized adaptive controllers design, followed by the stability analysis in Section 4. Finally, simulation results performed on double inverted pendulums are presented to illustrate the effectiveness of the proposed scheme.

2. Problem formulation

Consider an interconnected nonlinear time-varying system consisting of N subsystems described by

$$\begin{aligned}\dot{x}_{i,k} &= g_{i,k}(t)x_{i,k+1} + \theta_i^{\mathrm{T}}(t)f_{i,k}(\bar{x}_{i,k}) + \psi_{i,k}(y_1,\ldots,y_N,t),\\ \dot{x}_{i,n_i} &= g_{i,n_i}(t)u_i + \theta_i^{\mathrm{T}}(t)f_{i,n_i}(x_i) + \psi_{i,n_i}(y_1,\ldots,y_N,t),\\ y_i &= x_{i,1},\end{aligned} \tag{1}$$

where $i = 1,\ldots,N, k = 1,\ldots,n_i-1; x_i = [x_{i,1},\ldots,x_{i,n_i}]^{\mathrm{T}} \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are the states, input and output of the ith subsystem, respectively; $\bar{x}_{i,k} = [x_{i,1},\ldots,x_{i,k}]^{\mathrm{T}}$; the parameters $g_{i,k}(t) \in \mathbb{R}$ and $\theta_i(t) \in \mathbb{R}^{\nu_i}$ are unknown, bounded and piecewise continuous; $f_{i,k} \in \mathbb{R}^{\nu_i}$ are known smooth functions; and $\psi_{i,k} \in \mathbb{R}$ are unknown interactions among subsystems, which are locally Lipschitz in $y_1,\ldots,y_N$ and piecewise continuous in t. The full states of each subsystem are available for measurement.

For each subsystem, we make the following assumptions.

Assumption 1. The signs of $g_{i,k}(t)$, say, $sign(g_{i,k})$, are known and time-invariant, and $\mu_{i,k} = \inf_{t\geq 0}|g_{i,k}(t)| > 0$, where $k = 1,\ldots,n_i$.

Assumption 2. The unknown functions $\psi_{i,k},\ k = 1,\ldots,n_i$, satisfy

$$\psi_{i,k}^2(y_1,\ldots,y_N,t) \leq \sum_{j=1}^{N} \varrho_{i,k,j}\phi_{i,k,j}(y_j), \tag{2}$$

where $\varrho_{i,k,j} \geq 0$ are unknown constants and $\phi_{i,k,j}(y_j) \geq 0$ are known smooth functions.

Remark 1. It can be observed from (1) that all system parameters are allowed to vary with time and interactions exist in every equation of each subsystem, which significantly enlarges the system under consideration. In the case where $g_{i,k}$ and θ_i are limited to be constants, the system (1) was studied in Yoo et al. (2009) based on similar assumptions. However, in Yoo et al. (2009), to facilitate the disposal of interactions, only the output regulation problem was considered and the functions $\phi_{i,k,j}(y_j)$ in (2) were required to have the form $\phi_{i,k,j}(y_j) = y_j^2\bar{\phi}_{i,k,j}(y_j)$ with $\bar{\phi}_{i,k,j}(y_j)$ known smooth functions.

Remark 2. Assumption 1 implies that the virtual control coefficients $g_{i,k}(t)$ are either strictly positive or strictly negative with $g_{i,k}(t) \neq 0$. As a result, the system (1) is controllable. Note that $\mu_{i,k}$ are unknown constants and no restriction is imposed on the variation speed of the system parameters.

Based on the above assumptions, the objective is to design a decentralized adaptive backstepping control scheme such that all signals of the overall closed-loop system are bounded and the outputs $y_i(t),\ i = 1,\ldots,N$, track desired trajectories $y_{ri}(t)$ with prescribed tracking performance, where $y_{ri}(t)$ and their first n_i derivatives are known and bounded and $y_{ri}^{(n_i)}(t)$ are piecewise continuous.

The following lemma will be used in our design.

Lemma 1. *For any constant $\epsilon > 0$ and any variable $z \in \mathbb{R}$, the following relationship holds:*

$$0 \leq |z| - \frac{z^2}{\sqrt{z^2+\epsilon^2}} < \epsilon. \tag{3}$$

Proof. The first inequality of (3) is obvious. On the other hand, a simple calculation yields

$$|z| - \frac{z^2}{|z|+\epsilon} = \frac{\epsilon|z|}{|z|+\epsilon} < \epsilon, \tag{4}$$

which together with the fact that $\frac{z^2}{|z|+\epsilon} \leq \frac{z^2}{\sqrt{z^2+\epsilon^2}}$ gives the second inequality of (3). □

3. Decentralized adaptive controllers design

In this section, the decentralized adaptive controllers design is presented for the interconnected system (1). Before the backstepping design procedure, performance functions and error transformation are introduced for the purpose of guaranteeing prescribed tracking performance.