Mechanical Systems and Signal Processing 75 (2016) 589–606

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Harmonic amplitude dependent dynamic stiffness of hydraulic bushings: Alternate nonlinear models and experimental validation

Luke Fredette, Jason T. Dreyer, Todd E. Rook, Rajendra Singh *

Acoustics and Dynamics Laboratory, Smart Vehicle Concepts Center, Department of Mechanical and Aerospace Engineering, The Ohio State University, Columbus, OH 43210, USA

ARTICLE INFO

Article history:
Received 10 September 2015
Received in revised form
27 October 2015
Accepted 13 November 2015
Available online 17 December 2015

Keywords:
Hydraulic devices
Nonlinear system simulation
Multi-term harmonic balance method
Vehicle application
Sinusoidal testing

ABSTRACT

The dynamic stiffness properties of automotive hydraulic bushings exhibit significant amplitude sensitivity which cannot be captured by linear time-invariant models. Quasi-linear and nonlinear models are therefore proposed with focus on the amplitude sensitivity in magnitude and loss angle spectra (up to 50 Hz). Since production bushing model parameters are unknown, dynamic stiffness tests and laboratory experiments are utilized to extract model parameters. Nonlinear compliance and resistance elements are incorporated, including their interactions in order to improve amplitude sensitive predictions. New solution approximations for the new nonlinear system equations refine the multi-term harmonic balance term method. Quasi-linear models yield excellent accuracy but cannot predict trends in amplitude sensitivity since they rely on available dynamic stiffness measurements. Nonlinear models containing both nonlinear resistance and compliance elements yield superior predictions to those of prior models (with a single nonlinearity) while also providing more physical insight. Suggestion for further work is briefly mentioned.

1. Introduction

Hydraulic elastomeric devices are often employed in automotive powertrain and suspension systems because of their unique dynamic properties, leading to both vibration isolation and motion control [1–18]. These properties are achieved by an internal fluid system working in tandem with the elastomeric structure of a bushing. Despite some similarities with hydraulic engine mounts that have been extensively studied [1–9], the behavior of hydraulic bushings is quite different and merits its own in-depth studies [10–18]. Prior, though limited, investigations of these devices have largely focused on simpler transfer function type formulations based on the linear time-invariant (LTI) system theory [10–16]. For instance, Arzanpour and Golnaraghi [12] developed a reduced order linear system model for a hydraulic bushing which attempts to capture some aspects of the physics, such as fluid resistance, compliance, and inertance. Chai et al. [13–16] further developed the linear models for frequency and time domain characteristics for a laboratory bushing device and even introduced a nonlinear fluid resistance term [17]. Fredette et al. [18] recently developed a new laboratory experiment to measure the nonlinear fluid compliance of hydraulic bushing pumping chambers.

* Corresponding author. Tel.: +1 614 292 9044.
E-mail address: singh.3@osu.edu (R. Singh).

http://dx.doi.org/10.1016/j.ymssp.2015.11.017
0888-3270/

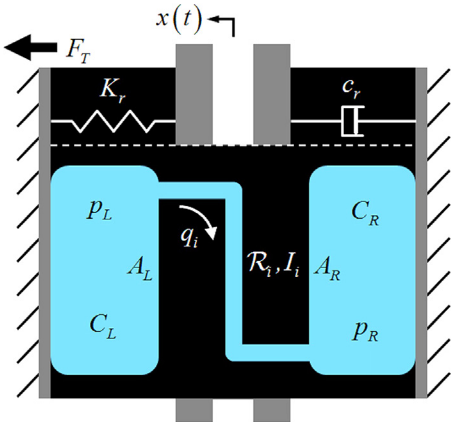

Fig. 1. Hydraulic bushing model. F_T is the transmitted force, $x(t)$ is the displacement excitation, K_r is the stiffness and c_r is the damping of the rubber path. For the pumping chambers, C is the fluid compliance, A is the effective pumping area, and p is the absolute pressure in each chamber, right (R) and left (L). In the inertia track, I_i is the fluid inertance, $\mathcal{R}_i$ is the fluid resistance and q_i is the volume flow rate.

Under harmonic excitation, hydraulic bushings exhibit significant amplitude dependent behavior [10,11,14–18], which cannot be described by the linear time-invariant system theory. Both the mechanical (rubber path) stiffness and fluid compliance elements of the bushings arise from the molded interfacial elastomeric material. Since most elastomeric materials exhibit inelastic behavior, nonlinear fluid compliance behavior has been suggested, [4,5,7–9,17,18] but the amplitude sensitivity has never been mathematically described. The chief goal of this article is therefore to propose new or improved quasi-linear and nonlinear reduced-order hydraulic bushing models, predict amplitude sensitivity characterization under harmonic loading, and experimentally validate alternate nonlinear models. Additionally, the prior work [17, 19, 20] on the multi-term harmonic balance method will be extended and refined to explain the underlying physics.

2. Problem formulation

Modeling of hydraulic bushings is challenging due to their complexity that arises due to the interacting nonlinear design features, nonlinear materials, and variation in production bushing designs. Hydraulic bushings are typically constructed of an elastomeric material constrained by a metal inner and outer sleeve, as shown in Fig. 1. Fluid-filled internal chambers deform when the bushing is displaced, pumping the fluid through a long passage between the chambers. A Kelvin–Voigt (linear system) model is assumed for the elastomeric structural path, while this article focuses on nonlinear fluid elements.

The lumped parameter modeling method is suitable for the fluid system contained within hydraulic bushings at low frequencies (up to 50 Hz), where the corresponding wavelength is much larger than the bushing dimensions. For the example case, model parameters include fluid compliance, C, and effective pumping area, A, for each chamber, the fluid resistance, $\mathcal{R}_i$, and inertance, I_i, of the single inertia track, and the stiffness, K_r, and viscous damping coefficient, c_r, of the rubber structure within the device. The state variables of the system include the absolute pressure, p, in each chamber (left (L) and right (R)) as well as the volume flow rate of fluid between the chambers, q_i. The outer sleeve is considered to be constrained, so the system is excited by the displacement of the inner sleeve, $x(t)$; the force transmitted to the outer sleeve, F_T, is the response.

Tractable lumped parameter models are needed to yield reasonable predictions of amplitude sensitive dynamic stiffness, which is a useful metric for design and diagnostic purposes, and to provide physical insight. Accordingly, the specific objectives of this article are as follows. (1) Propose and experimentally validate quasi-linear and nonlinear bushing models (with nonlinear resistance and compliance elements) which capture amplitude sensitivity in the example case of a production bushing with a single inertia track as displayed in Fig. 1. (2) Refine and utilize the semi-analytical multi-term harmonic balance method (MHBM) to construct dynamic stiffness spectra and gain physical insight into the role and interaction of nonlinearities in the model. The scope of the work is limited to sinusoidal excitation only with peak-to-peak displacement amplitudes of 0.1 mm, 0.5 mm, 1.0 mm, and 2.0 mm over a frequency range from 1 to 50 Hz to capture the amplitude sensitivity of the tuned dynamic properties.

The bushing is characterized by a cross-point dynamic stiffness, assuming a sinusoidal excitation at angular frequency Ω, where x_m is the mean component and x_a is the peak-to-peak excitation amplitude,

$$x(t) = x_m + \frac{x_a}{2}\sin(\Omega t). \tag{1}$$

The force transmitted to a rigid base is calculated by summing the contributions from parallel structural and fluid paths. The force through the elastomeric structural path may be directly calculated, $F_r = K_r x + c_r \dot{x}$, while the force through the fluid