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Brief paper

Fault recoverability and fault tolerant control for a class of interconnected nonlinear systems[☆]

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ABSTRACT

This paper considers a class of interconnected nonlinear systems with reconfigurable physical couplings. Under decentralized control laws, a fault recoverability condition is provided which reveals the capability of the interconnected system to tolerate the faults under given couplings. Consequently three fault tolerant control methods are proposed that rely on both control reconfiguration and coupling topology reconfiguration. The couplings' effect on system performance is analyzed from the overall system point of view. An example for a system with three pendulums is taken to illustrate the theoretical results.

1. Introduction

Fault tolerant control (FTC) aims at guaranteeing the system goal to be achieved in spite of faults (Blanke, Kinnaert, Lunze, & Staroswiecki, 2006; Jiang, Staroswiecki, & Cocquempot, 2006; Tao, Chen, Tang, & Joshi, 2004; Zhang & Jiang, 2008). A fault is labeled as *recoverable* if there exists a control law such that the post-fault system satisfies the design specifications (Staroswiecki, 2008; Wu, Zhou, & Salomon, 2000; Zhang & Jiang, 2003). For non-decomposable specifications, the FTC design of an interconnected system must be considered for the overall system rather than any individual subsystem (Patton et al., 2007). For an interconnected system, two main kinds of couplings among subsystems can be considered (Patton et al., 2007): C1. *Physical couplings* where all couplings are physical and are independent from the individual control law of each subsystems, and thus cannot be changed or removed arbitrarily; C2. *Network connections* where each subsystem communicates with others through network, e.g., multi-agent systems. Interactions among subsystems are represented through the control channels of each subsystem, and thus can be designed and changed.

For system with C1 coupling, the FTC law of each subsystem often consists of two parts, see Gandhi and Mhaskar (2009), Panagi and Polycarpou (2013), Panagi and Polycarpou (2011), Patton et al. (2007) and Tong, Huo, and Li (2014): one is for the FTC of self-dynamics, and another one compensates for the coupled-dynamics. This leads to a distributed control structure where the coupling term is treated using a robustness approach. There are two limitations behind these methods: (1) exchange of states information would increase communication burdens and bring some uncertainties, e.g. delay, also for complex nonlinear structure, the coupling term may not be compensated for easily; (2) the couplings' effect is analyzed locally in each subsystem, this might not reveal effectively the relations between couplings and FTC performance of the overall system. Moreover the reconfigurable physical coupling that might be present in some practical systems is also not considered.

For system with C2 coupling, FTC methods utilize the coupling's effect actively and globally, i.e., when one subsystem is faulty, both the coupling topology and control laws of other subsystems are reconfigured, see Semsar-Kazerooni and Khorasani (2008), Staroswiecki and Moradi Amani (2014), Tousi and Khorasani (2012) and Yang, Staroswiecki, Jiang, and Liu (2011). It can be seen that such a FTC idea emphasizes more the importance of coupling effect than that for C1 coupling.

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This paper considers a class of interconnected nonlinear systems with C1 coupling that may be reconfigurable. Inspired by the FTC idea for systems with C2 coupling, a fault recoverability condition and three FTC methods relying on both control reconfiguration and coupling topology reconfiguration are proposed based on the cyclic-small gain approach that was recently developed in Jiang and Wang (2008) and Liu, Hill, and Jiang (2011). Such an approach is very general and helps to provide a decentralized control structure (each subsystem's control law only uses its own states which does not compensate for the coupled-dynamics) and to deeply analyze the coupling's effect on fault recoverability and FTC from the overall system point of view.

In the rest of the paper: Section 2 provides a decentralized control method, Sections 3 and 4 address the issues of fault recoverability and fault tolerant control respectively, Section 5 gives an illustrative example followed by some concluding remarks in Section 6.

2. Decentralized control design

A directed graph (digraph for short) is denoted as $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \ldots, n\}$ is the set of nodes and $\mathcal{E}$ is the set of arcs, $(j, i) \in \mathcal{E}$ denotes an arc from node j to node i. A path in $\mathcal{G}$ from node i_0 to node i_k is a sequence of arcs $(i_0, i_1)(i_1, i_2)\cdots(i_{k-1}, i_k)$, where nodes $i_\iota \in \mathcal{N}$ and arcs $(i_\iota, i_{\iota+1}) \in \mathcal{E}$, $\iota = 0, 1, \ldots, k-1$, $k \geq 1$. The connection behavior of an interconnected system is described by a digraph $\mathcal{G}$, where node i models subsystem i, an arc (j, i) indicates that subsystem j is a *neighbor* of subsystem i in the sense that subsystem i is coupled with subsystem j. Also suppose that there exist n candidate topologies denoted as $\mathcal{G}_i$, $i \in \mathcal{N}$. Denote $N_j(i)$ as the set of neighbors of subsystem i under topology $\mathcal{G}_j$.

Consider an interconnected system with m subsystems and n candidate coupling topologies. The dynamics of subsystem i under topology $\mathcal{G}_k$ takes the form:

$$\dot{x}_i = g_{0i}(x_i) + g_i(x_i)u_i + \sum_{j \in N_k(i)} h_{ij}(x_j), \quad i \in \mathcal{M} \tag{1}$$

where $\mathcal{M} \triangleq \{1, 2, \ldots, m\}$, $x_i \in \Re^{n_i}$ is the measurable state, $u_i \in \Re^{p_i}$ is the control input. $g_{0i} + g_i u_i$ represents the *self-dynamics* with g_{0i} and g_i being smooth nonlinear functions. $h_{ij}(x_j)$ represents the *coupled-dynamics* w.r.t. subsystem j, $h_{ij} = 0$ if $j \notin N_k(i)$, $h_{ij}(x_j)$ is Lipschitz with $h_{ij}(0) = 0$, i.e., $|h_{ij}(x_j)| \leq l_{ij}|x_j|$ for $l_{ij} > 0$.

Assumption 1. There exist smooth, proper, and positive definite functions $V_i : \Re^{n_i} \to \Re$ and constants $a_i > 0$, $b_i > 0$, $d_i > 0$, $\lambda_i > 0$, such that $\forall i \in \mathcal{M}$

$$\begin{aligned} &a_i|x_i|^2 \leq V_i(x_i) \leq b_i|x_i|^2 \\ &\left|\frac{dV_i}{dx_i}\right| \leq d_i|x_i| \\ &\inf_{u_i \in \Re^{p_i}} \left\{\frac{dV_i}{dx_i}(g_{0i} + g_i u_i) + \lambda_i V_i\right\} < 0, \quad \forall x_i \neq 0. \end{aligned} \tag{2}$$

For each $\varepsilon > 0$, $\exists \delta > 0$ such that if $|x_i| < \delta$, $x_i \neq 0$, then $\exists u_i$ with $|u_i| < \varepsilon$ such that $\frac{dV_i}{dx_i}(g_{0i} + g_i u_i) \leq -\lambda_i V_i$.

Assumption 1 implies that without coupling, each subsystem has a *control Lyapunov function* satisfying the *small control property* (Sontag, 1989). According to the universal formula for stabilization, the decentralized state-feedback control law u_i, $i \in \mathcal{M}$, is designed as:

$$u_i(x_i) = \begin{cases} -\dfrac{\Lambda_1 + \sqrt{\Lambda_1^2 + |\Lambda_2|^4}}{|\Lambda_2|^2}(\Lambda_2)^\top & \Lambda_2 \neq 0 \\ 0 & \Lambda_2 = 0 \end{cases} \tag{3}$$

where $\Lambda_1 \triangleq \frac{dV_i}{dx_i} g_{0i} + \lambda_i V_i$, $\Lambda_2 \triangleq \frac{dV_i}{dx_i} g_i$. Consequently,

$$\frac{dV_i}{dx_i}(g_{0i} + g_i u_i) \leq -c_i|x_i|^2$$

where $c_i \triangleq \frac{\lambda_i}{a_i}$. For a fixed V_i, the value of c_i depends on λ_i that is often impossible to be designed arbitrarily large. There exists an admissible u_i with maximal $\lambda_i^{\max}$ under which a maximal decay rate $c_i^{\max}$ w.r.t. V_i is reached.

The time derivative of V_i along the solution of (1) is

$$\begin{aligned} &\dot{V}_i \leq -(c_i - \theta_i)|x_i|^2, \quad 0 < \theta_i < c_i \\ &\forall |x_i| \geq \frac{\sum_{j \in N_k(i)} d_i l_{ij}|x_j|}{\theta_i}. \end{aligned} \tag{4}$$

Note that inequality (4) can be derived from

$$V_i \geq \max_{j \in N_k(i)} \left(\frac{b_i(m_{ki}d_i l_{ij})^2}{a_j \theta_i^2} V_j\right)$$

where m_{ki} is the number of subsystems in $N_k(i)$. It can be seen that if $\theta_i = c_i - \Delta$, where Δ is an infinite small number, then the gain from V_j to V_i is minimal. Define

$$\gamma_{ij} \triangleq \frac{b_i(m_{ki}d_i l_{ij})^2}{a_j(c_i)^2}, \qquad \gamma_{ij}^{\min} \triangleq \frac{b_i(m_{ki}d_i l_{ij})^2}{a_j(c_i^{\max})^2}.$$

Theorem 1. *Under Assumption 1, there exist decentralized control laws $u_i(x_i)$, $\forall i \in \mathcal{M}$, such that the interconnected system (1) is asymptotically stable at the origin if*

$$\gamma_{i_1 i_2}^{\min}\gamma_{i_2 i_3}^{\min}\cdots\gamma_{i_r i_1}^{\min} < 1 \tag{5}$$

for all $i_j \in \mathcal{M}$, $i_j \neq i_{j'}$ if $j \neq j'$.

Condition (5) shows the couplings' effect on the stability of the whole system rather than any individual subsystem as in Panagi and Polycarpou (2011), which means that the composition of the minimal gains along every cycle is less than 1. Such a condition depends on two factors: the cycles in the coupling topology and individual control design. In each cycle, it allows for large gains of some subsystems (resulting from small decay rate or large coupling parameters) being compensated by small gains of other subsystems (resulting from large decay rate and small coupling parameters). Not all gains but only gains in each cycle need to be considered, the number of cycles depends on the coupling topology $\mathcal{G}_k$. Therefore Condition (5) is not restrictive and can be achieved by both individual control design and coupling topology design. Such a condition reveals a trade-off that should be achieved among all subsystems, and relaxes the design in each individual subsystem. It even disappears for the system without cycle in the coupling topology, e.g. the tree structure.

In order to prove Theorem 1, the following lemma is given which can be obtained straightly from the cyclic small gain theorem in Liu et al. (2011).

Lemma 1. *Consider an interconnected system $\dot{x}_i = \zeta_i(x)$, $i \in \mathcal{M}$, where $x_i \in \Re^{n_i}$, $x = [x_1^\top, \ldots, x_m^\top]$, ζ_i is continuous and locally Lipschitz. If for each subsystem i, there exists a function $V_i : \Re^{n_i} \to \Re_{\geq 0}$ that satisfies (2) and $V_i(x_i) \geq \max_{i \in \mathcal{M}-\{i\}}(\gamma_{ij}(V_j(x_j))) \Longrightarrow \frac{dV_i}{dx_i}\zeta_i(x) \leq -\iota_i V_i(x_i)$, $\iota_i > 0$, then the interconnected system is asymptotically stable at the origin if $\gamma_{i_1 i_2}\gamma_{i_2 i_3}\cdots\gamma_{i_r i_1} < 1$ for all $i_j \in \mathcal{M}$, $i_j \neq i_{j'}$ if $j \neq j'$.* □